For a Constructive Post-Modernity: The Post-Modern Transcendental of Language and the Phenomenology

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Abstract.

In this contribution, I suggest to interpret our present age, signed by deep changes in every realm of the society, and of the culture, as a Post-Modern Age, not in the usual nihilist interpretation of Post-Modernity, but in a constructive way. This can more easily emphasized if we see at the three ages, Ancient, Modern, and Post-Modern, from the "transcendental" standpoint. That is, from the standpoint of the ontological foundation of truth, so to characterize the Ancient Age with the "Transcendental of Being", the Modern age with the "Transcendental of Knowing", and the Post-Modern Age with the "Transcendental of Language", by developing Otto Apel's early suggestion (Apel). The original part of my contribution consists, indeed, in emphasizing that it is not possible to understand Peirce's semiotic turn in its epochal relevance, if we do not consider also Peirce's fundamental contribution to the foundations of modern formalized logic and mathematics, with applications also to the modern mathematical physics. Peirce's contribution consists, indeed, in the proposal of an "algebra of relations", correcting in a semiotic/semantic way the early formalistic proposal of an "algebra of logic" by Ernst Schröder, without any necessary reference to a knowing, conscious subject. Edmund Husserl also shared this same criticism, almost in the same years, but independently from Perice's semiotics, because Husserl criticized Schröder formalistic approach to algebraic logic from the standpoint of the Transcendental of Knowledge, i.e., the subject-object intentional relation, proper of phenomenology. Therefore, in the First Section, we give a short presentation of the nihilist interpretation of Post-Modernity, and an examination of Jürgen Habermas' deep criticism of this nihilist interpretation, precisely in the light of Peirce's com-

plete linguistic turn. Afterwards, we develop in the Second Section, our comparison between the phenomenological ontology, and the semiotic ontology in the light of John Poinsot's pro-to-semiotics. This is precious for distinguishing at the down of Modernity between "object" and "thing", i.e., the main unsolved issue of the phenomenological ontology at the end of Modernity, from the standpoint of a nascent logic of relations, and then for understanding Peirce's semiotic realism. Therefore, in the Conclusions, I propose a semiotic triangulation, different from Habermas' one, of the "first-person language" (*I-We talk*) of intentional sub-ject(s), in terms of the two "third-person languages" of external observers ($O_{1\text{-}2}$ *talk*), follow-ing the rules of extensional (mathematical), and intensional (philosophical) formalized logics, without any pretension of "naturalization" / "simulation" of the former by the latter ones.

1 Toward a post-modern philosophy: the linguistic issue

1.1 The present age as a Post-Modern Age and the place of philosophy in it

We are living today an age of deep mutations in the social, political, economic, and cultural components of our lives. Task of philosophers is the interpretation of these changes, starting from the evidence that they are today involving also the role of the philosophy itself, particularly as to the relationship between sciences and humanities, and as to the relative collocation of the philosophical thought. During the Modern Age, indeed, starting from Descartes' methodological distinction between natural and mathematical sciences, on one hand, and the philosophical disciplines devoted to the analysis of consciousness contents, on the other one, philosophy found its own stable collocation among humanities, also in the Academic structure of our Universities. Descartes' seminal distinction had its full development through the systematic development of the phenomenological method from the beginning of the last century, giving philosophy, by Husserl's *epoché* as to the naturalistic thesis, its own logical and epistemological statute of a rigorous "science of essences" (Husserl, *Philosophie als strenge Wissenschaft*), with respect to the natural and mathematical sciences. Two challenges, however, today attack such an armistice in the modern confrontation between science and philosophy.

Firstly, the newborns among the modern natural sciences, i.e., the "cognitive sciences" and especially the "cognitive neurosciences", vindicate a growing role in the natural explanation of the subjective conscious states and acts, "intentionality" and "inter-subjectivity" included[1]. Because of their success, they gave an essential impulse to the research program of

[1] These developments depend on the change of paradigm in cognitive neurosciences from the early "symbolic approach", related to the functionalist approach to the cognitive sciences of the Artificial Intelligence (AI) research program inaugurated at the famous

the so-called "naturalization of phenomenology", inaugurated by J. Petitot and F. J. Varela during the late 90's of the last century (Petitot, Varela e Pachoud). Anyway, what is important to emphasize is that the cognitive sciences generally do not pursue a reductionist approach as to the conscious states and acts, like the early behaviorist approach in neurosciences. On the contrary, they suggest a methodic "triangulation" between the subjective reports "in first person" (*I/We-talk*) of the conscious states/actions, and two objective correlates of them, as to "external observers" (*O-talk*). They are the neurophysiological modifications of the brain dynamics, as implementing – and this is the second objective correlate of conscious states – an "information processing" of the stimuli from the environment, from the other humans included (see (Gardner); (Basti, *Intentionality and Foundations of Logic*; Basti, *A Formal Approach to the Ontology of Social Beliefs*)). This suggests the notion of the "extended mind", i.e., the location of mind not "inside the brain", but into the dynamic/informational interface between the brain, its body and its environment, the social environment included (see (Marturana e Varela); (Bateson); (Clark); (Noë)). This mind localization as including the body is common, however, to Aquinas' theory of the intentional mind (Basti, *Intentionality and Foundations of Logic*), and to many thinkers of the phenomenological school as well – think only at M. Merlau-Ponty and at his theory of the "bodily schemes" (Merlau-Ponty).

Secondly, the other attack to the armistice in the modern confrontation between philosophy and science is from the inside of the same philosophy. Indeed, in front of a progres-

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Dartmouth Conference of 1956 (Gardner), to the pre-symbolic and hence "intentional approach". This latter was developed during the last twenty years essentially by W. Freeman and his group at the University of Berkeley (see (Freeman, *How brains make up their minds*) and (Freeman, *Intentionality*)) and by G. Rizzolatti and his group at the University of Parma (see (Rizzolatti, Fadiga e Gallese) and (Ortigue, Sinigaglia e Rizzolatti)). For our aims, it is important to emphasize that Freeman's approach developed itself within the "ontological" approach to intentionality originating from Aquinas' theory in the Middle Age, reinterpreted according to M. Heidegger's (Heidegger, *The Basic Problems of Phenomenology. Revised Edition.*) and M. Merlau-Ponty's (Merlau-Ponty) criticism to Husserl's theory (see (Freeman, *Intentionality*) and (Freeman, *Nonlinear dynamics and the intention of Aquinas*)). On the contrary, Rizzolatti's approach is strictly related to E. Stein's theory of the "intersubjective intentionality" (*Einfühlung*) (see (Gallese), and (Manganaro)).

sive desertification of the Departments of Philosophy inside the Faculties of Humanities of our Universities, we assist to a parallel blooming development of philosophical researches within the Departments of Computer Science, overall in Europe. These researches are generally collected under the comprehensive names of "philosophical logic" (Burgess), and "formal philosophy" ("formal ontology", "formal epistemology", "formal ethics", etc.: see for an introduction (Hendricks and Symons)). On this regard, it is significant the following quotation from the *Introduction* to Second Edition (2002) of the monumental *Handbook of Philosophical Logic,* arrived this year at its 17th volume, by its Chief Editor, D. M. Gabbay:

> The researcher in this area is having more and more in common with the traditional philosopher who has been analyzing such questions for centuries. […] I believe the day is not far away in the future when the computer scientist will wake up one morning with the realization that he is actually a kind of formal philosopher (Gabbay and Guenthner, vol. 1, viii-ix).

Effectively, Gabbay had to wait only ten years for seeing the concrete realization of such a visionary approach to formal philosophy. In 2012, indeed, a new study program in "Computer Science and Philosophy", with the relative BA and MA degrees, started at Oxford University (http://www.ox.ac.uk/admissions/undergraduate/courses-listing/computer-science-and-philosophy), followed henceforth by all the main UK Universities. Similar study programs, with different denominations, are actually present in several USA, Japanese, and European Universities, and in the very next future, also in Indian Universities.

1.2 The Post-Modern Age as the Communication Age

1.2.1 The nihilist interpretation of "post-modernity"

The growing success of such an approach has a double, *theoretical* and *social* explanation. Let us start with the social one, because this immediately justifies the title of this paper, that is, the reference to the designation of our "Information Age" or "Post-Industrial Age", or "Semiotic Age", as a "Post-Modern Age". Today, indeed, "communication" and

"knowledge" interrelate in an inextricable way, changing our *private* and *public* "life-world" – the *Lebenswelt* in Husserl's and Habermas' conceptions, respectively –, as it never happened in the past. The denotation of our age as "Post-Modernity" in social philosophy generally refers to the seminal work of J. F. Lyotard on the "post-modern condition" (Lyotard), and to the fierce debate it triggered, before all in social philosophy (Habermas, *Modernity versus Postmodernity*).

Generally, indeed, what is prevailing is the "nihilist interpretation of post-modernity". According to S. Benhabib's critical synthesis, this extreme interpretation of post-modernity characterizes itself as (Benhabib 18):

1. *Anti-foundational*: the refusal of the "transcendental subjectivity" as foundation of truth in logic, and goodness in ethics;
2. *Anti-historical*: the refusal of the modern notions of "progress" and "history";
3. *Anti-metaphysical*: the refusal of the notions of "metaphysics" and of "absolute truth", as far as based on a unitary "being", beyond particularity and change.

Before Benhabib, the Italian Philosopher G. Vattimo, in a collection of essays with the significant title *The End of Modernity* (Vattimo), explicitly connected such an interpretation of the Post-Modern Age to Nietzsche's and Heidegger's nihilist position, as the necessary outcome of the modern transcendentalism. Following M. Heidegger's synthesis, Vattimo individuates in the "onto-theological principle", characterizing a large part of Middle-Age metaphysics and theology, the ultimate root of the Post-Modernity nihilist stance, even though with the mistake of involving also Aquinas among the onto-theologians (Basti, *L'idea di scienza di Maritain fra passato e futuro*). As Heidegger synthesized in his book *On the Essence of Truth* (Heidegger, *Vom Wesen der Wahrheit*)[2], the "onto-theological principle"

[2] This book derives from the text of a conference held for the first time by Heidegger in 1930, and many times repeated till 1932 under the same title, even though published for the first time only in 1943. Generally, the scholars consider it as the first text of the so-called Heidegger's *Kehre*. It is ultimately, indeed, a sort of theoretical introduction of the two monumental monographies on

consists in affirming the necessary supposition of the existence of God as foundation of any true rational knowledge, before all in ontology. In this way, evidently, the existence of God cannot be in principle the final outcome of any rational path of knowledge. Therefore, it must be supposed "by faith", and therefore ultimately by *an act of will,* not of reason, so to unveil the *voluntarist* character of any modern transcendentalism, as A. Schopenhauer first stated, and F. Nietzsche re-proposed, in their criticism of the Kantian and of the Hegelian transcendentalism. Of course, in such a framework, all *faiths* either religious or atheistic, intended as the necessary start points of any rational activity of humans, become equivalent, so to justify Heidegger final statement that "the essence of truth is freedom". On the other hand, and more radically, all this justified Nietzsche's "reduction of logic to rhetoric", given that "what is essential is not that something *is* true, but *to hold something for true*" (Nietzsche 77-78).

Therefore, in our communication age where communicating is the supreme value – "if you do not communicate, you do not exist" –, G. Vattimo states that the post-modern nihilism consists in a void, indefinite discussing about values, with the ultimate supposition of their universal equivalence, where the *Grund* has become ultimately the absolute *Ab-Grund*.

To sum up, if the nihilism consists in the systematic "reduction of being to value", of "what is" to "what we want to be", this reduction in our Post-Modern "Information Age", according to G. Vattimo, becomes the value of the "indefinite communication exchanges", so that today,

> The nihilism does not consist in the fact that the being is in the power of a subject, but in the fact that being is completely dissolved into the discussion about values, in the indefinite transformations of the universal equivalence (Vattimo 29-30).

1.2.2 Habermas' criticism of "post-modernity"

This interpretation of post-modernity, according to J. Habermas' criticism, justified

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Nietzsche thought published by Heidegger between 1936 and 1946

indeed several conservative, *antimodernist* positions both in politics and in economics. Effectively, Habermas distinguishes among several antimodernist positions. They are, essentially, the aesthetical position of the "young conservatives" (e.g., J. Derrida); the neo-Aristotelian position of the "old conservatives", (e.g., R. Spaemann), and, finally, the ultra-liberalist positions of the "neo-conservatives". Against these "interested" positions, Habermas proposes that, "instead of giving up modernity and its project as a lost cause, we should learn from the mistakes" (p. 11), to correct what in Modernity got wrong, without losing its undoubtable merits, overall in the political, social, and ethical realms.

This is precisely what I am intending here with the defense of the work program of a "Constructive Post-Modernity": a sort of "synthesis/overcoming", in a neo-Hegelian sense, of the mistakes both of the Classical ("thesis"), and of the Modern ages ("antithesis"), by learning from these mistakes. This can help us to solve our actual problems and terrible challenges – think only at ecology and terrorism – in an open-minded way, against the obsolete polemical opposition modern/conservative. This concerns the foundational questions in ontology, epistemology, and ethics, and therefore also in politics and economics. To continue to deal with them according to the old modern categories (think only at the opposition "left / right" in politics) is out of date.

Effectively, in the light of formal philosophy, just J. Habermas' *foundational theory of truth* is an example according to me of such a "constructive interpretation of post-modernity"[3]. This theory constitutes, indeed, the core of his collection of philosophical essays, *Wahreit und Rechtfertigung,* and specifically the core of its introductory essay about ontology, interpreted in the framework of "formal pragmatics" (Habermas, *Einleitung: Realismus nach der sprachpragmatischen Wende*). Not casually, indeed, the other fundamen-

[3] For this reason, and for a kind license of the Author and of the Suhrkamp Publisher, I inserted this essay of Habermas, given into a new Italian translation by R. Giovagnoli, within a collection of papers about different approaches to "formal ontology", including also, among others, papers of J. Searle, and A. Ales Bello (Basti and Mobeen, Ontologia Formale).

tal essay of this collection is about Habermas' criticism to the "transcendental philosophy", that is, about the core of the modern foundational theory of truth, as far as based on knowledge, and not on language (Habermas, *Wege der Detranszendentalisirung. Von Kant zu Hegel and zurück*).

Because of formal pragmatics, indeed, it is possible, according to Habermas, to propose a new "triangulation" as to the fundamental problems of meaning and of predication in philosophical logic versus the mathematical one, so to help the post-modern philosophy to exit from the "modern jails" of the transcendentalism and mentalism, in dealing with the modern challenges between philosophy and science. The third vertex of the triangle, according to the effective image used by Habermas is the *alterity* (second-person talk), after the two ones of the *subject* (first-person talk), and of the *object* (third-person talk) of the modern transcendentalism. This transforms the edges of the knowledge triangle in those of the *communication processes* of pragmatics, within which "truth" progressively emerges as a social construct, in a "spiral" *never-ending*, and *progressive* process. This foundational scheme is very effective for our times, in which "we" (as plural first-person) – the Western, European civilization – are constrained to reckon with the "you" (as plural second-person) of the other civilizations, their philosophies and their religions included. Effectively, this is one of the main objects of Habermas' last productions.

In this way, it is possible to understand all the relevance of the statement that Habermas put at the beginning of his quoted essay on ontology:

> Once Frege replaced the mentalistic *via regia* of analyzing sensations, representations, and judgments with a semantic analysis of linguistic expressions and Wittgenstein radicalized the linguistic turn into a paradigm shift, Hume and Kant's epistemological questions could have taken on a new, pragmatic significance. In the context of lived practices, of course, *they have lost their primacy over questions in the theory of communication and action*. Yet even, within philosophy of language, the traditional order of explanation has persisted. As ever, theory takes precedence over practice, representation over communication; and the semantic analysis

> of action depends on a prior analysis of knowledge (Habermas, *Einleitung: Realismus nach der sprachpragmatischen Wende* 7. Italics mine).

Against this *incomplete* linguistic turn, still characterizing the analytic philosophy following G. Frege's and L. Wittengstein's approaches, Habermas vindicates the complete linguistic turn characterizing Ch. S. Peirce's semiotics, and the primacy in it of pragmatics as to syntax and semantics in logic. Effectively, Peirce's *algebraic* "logic of relations" is both theoretically and temporally preceding Frege's and Wittengstein's non-algebraic logistics, still depending on a "conceptualist" modern bias, that is, on *a primacy of knowledge over language* and over linguistic practices. This is evident before all in Frege's systematic attempt of foundation of arithmetic over "classes" – i.e., where numbers are "classes of classes" – instead of over "sets". He pursued, indeed, the vain hope of escaping in such a way G. Cantor's set theory antinomies in the foundations of mathematics, a research program dramatically frustrated by B. Russell's discovery of the famous "Russell Antinomy". The "conceptualist bias" characterizing the incomplete linguistic turn in Frege's mathematical logic is evident since the title of his masterpiece *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens* (1879) (Frege), and formally it is evident by his *axiom schema of comprehension* for the foundation of classes as linguistic counterpart of mental concepts.

So, coming back to Habermas' last quotation, if Frege's mathematical logic is an attempt of giving a linguistic counterpart of Kant's formal conceptualism, Wittengstein's philosophy of language is a counterpart of Hume's empirical conceptualism, given that the "atomic propositions" of his logical construction are as many linguistic counterparts of Hume's "raw feels". In both cases, however, this emphasizes the incompleteness of Frege's and Wittengstein's "linguistic turn" because in the philosophy of language based on these principles "as ever, theory takes precedence over practice, representation over communica-

tion; and the semantic analysis of action depends on a prior analysis of knowledge". I.e., they move still inside the "Modern Transcendental of Knowledge", and not yet completely inside the "Post-Modern Transcendental of Language".

1.3 The Post-Modern Age as the Semiotic Age, and the completion of the linguistic turn

1.3.1 Peirce's completion of the linguistic turn and post-modernity

Let us now introduce the second "theoretical" motivation justifying the denotation of present time as a "Post-Modern Age". The just remembered "incompleteness" of the linguistic turn in Frege and Wittengstein justifies, indeed, the position of the American philosopher and semiotician J. Deely in defining our "Communication/Information Age" as a "Post-Modern Age", as far as based on the "complete linguistic turn" of Ch. S. Peirce. This is the main thesis of Deely's monumental book about the history of philosophy, defined by his author – and this is the sub-title of the book – as "the first postmodern survey of philosophy from ancient times to the turn of the twenty-first century" (Deely, *Four Ages of Understanding*). The complete linguistic turn, as far as based on the foundational primacy of "communication" and "action", over "knowledge" and "consciousness" depends indeed on Peirce's philosophy. Therefore, Deely's approach supports my choice of inserting also Habermas' social philosophy and ontology, aimed at correcting the mistakes of the Modern Age without renouncing to its positive contributions, into the file cabinet of "the constructive post-modern philosophy", overall because Habermas' position explicitly depends on Peirce's complete linguistic turn.

However, Deely's semiotic characterization of our twenty-first century philosophy concerns also – far beyond his intention and expertise, but completely in continuity with the intention and expertise of his master, C. S. Peirce – one of the main novelties of our actual philosophical panorama. Namely, it concerns the *philosophical logic* and the *formal philoso-*

phy, as distinguished but strictly related to the *mathematical logic* and to the *theoretical computer science* (TCS). They are, indeed, deeply changing the modern way of interpreting the relationship between the pure and applied mathematical sciences, on one hand, and the different philosophical disciplines, on the other one.

Indeed, Peirce's threefold analysis (syntax, semantics, pragmatics) of language as a system of signs in semiotics characterizes also the theory and the practice of the modern formal logic, because ultimately deriving from C. S. Peirce fundamental contributions to the *algebra of relations* (Peirce, *The logic of relatives*) and to the *algebra of logic* (Peirce, *On the algebra of logic*). These contributions are nowadays involving, through the so-called "Category Theory" (CT), the same foundations of logic, mathematics, computability theory, and the same fundamental (quantum) physics, quantum computing included (Basti, *From formal logic to formal ontology*).

On the other hand, it is impossible to speak about the "complete linguistic/semiotic turn" as an "epochal shift" between a Modern and a Post-Modern Age, if this turn does not involve also the foundations of modern mathematics and physics. The *algebra* and the *algebra of relations* are, indeed, the only possible formal link between logic and language on one hand, and mathematics and physics, on the other hand, as the same Descartes and Leibniz anticipated at the down of the Modern Age, but, in the last two centuries, we often forgot[4].

1.3.2 The linguistic turn, and the distinction between axiomatic mathematical logic and axiomatic philosophical logic

Effectively, also the main distinction in contemporary formal logic after the linguistic

[4] I. H. Anellis, in a comprehensive study about the reciprocal acceptance of the respective works in logic between C. S. Peirce and B. Russell (Anellis), emphasized that the late recognition of the relevance of C. S. Peirce work on the foundations of mathematical logic, is due essentially to B. Russell. He, indeed, overall in the *Principia,* did not appreciate Peirce logical work, even though generally was appreciating him as a philosopher. For an update and complete synthesis of Peirce's work relevance in the history of mathematical logic, see (Brady).

turn, between "mathematical logic" and "philosophical logic", depends on these distinctions[5]. Indeed, while in mathematical logic, the formal analysis of the scientific language as far as based on pure and applied mathematics, limits systematically itself to the "syntactic" and "semantic" components of such a language, the "philosophical logic" goes further. It extends the formal analysis also to the "pragmatic" component of the ordinary languages that are the languages in which the philosophical doctrines are ordinarily expressed.

Such languages, indeed, are like as many "implicit ontologies", related with the "beliefs" – or "plural first-person" intentional statements (*we* believe that…) of groups and peoples. This depends on the fact that necessarily the different, traditional, philosophical doctrines express themselves into the different ordinary languages, as far as related to different cultures. Therefore, the formalization of these doctrines by using the philosophical logic can be, if used and diffused in the philosophical realm, and not only in the computer science realm, an essential means for facilitating the intercultural dialogue. This formalization can make possible, in our twenty-first century, the *globalization* of the philosophical cultures and of their richness on a rigorous and unambiguous basis, without negating their differences, but on the contrary, preserving them. the formalization of mathematics, from the publication of the *Principia Mathematica* on, made possible with the scientific culture. It allowed the globalization of the scientific culture starting from the last century, without negating the differences among the theories. On the contrary, the formalization preserves the differences, while suggesting new possible connections, so to promote continuously new solutions of open problems, because formalization makes them *universally and unambiguously* available to the

[5] It is not casual that C. I. Lewis pioneering work on the philosophical logic by his axiomatization of modal logic, is based on a deep criticism of the notion of "material implication" in the mathematical logic of *Principia.* Overall, if we apply it to the notion of "implication" in philosophical logic, before all in metaphysics. This is, on the contrary, precisely the fundamental mistake that is at the basis of Wittengstein's *Tractatus*, effectively published, with an enthusiastic preface, by Russell himself, and not by Wittgenstein, who evidently did not know at that time the work of Lewis. Not casually, the same C. I. Lewis, differently from Russell (see preceding note), dedicated a paragraph to Pierce in his early book on the axiomatic modal logic, because "the contributions of C.S. Peirce to symbolic logic are more numerous and varied than those of any other writer — at least in the nineteenth century" (Lewis, *A Survey of Symbolic Logic* 79).

scholar inquiry, and no longer imprisoned in their linguistic/cultural/belief jails[6].

On the other hand, formal pragmatics, inside the more general framework of the philosophical logic, concerns also the computational linguistics and the computational logic. Therefore, it requires an extension of the same set-theoretic formal semantics also in mathematical logic, so to include the "non-standard" set-theories and the same modal logics in their coalgebraic interpretation, with the related notion of "local truths". They characterize, indeed, the *practical and applied inferences* in non-standard set theory and formal semantics, as opposed to the "logical truths", characterizing human abstract way of thinking in the *speculative reasoning*, in standard set theory and formal semantics (see below Sect. 3).

Also in this case, however, we can find in Peirce's work a pioneering idea. Effectively, after declared himself as a "Realist", because affirming the reality of "generals" (i.e., of the "natural kinds" as denotata of common names, like "animal" or "horse") at the very beginning of his research (Peirce, *Nominalism versus Realism*), Peirce afterwards offers a definition of the modal operator of "possibility" in *pragmatic* terms. He defines it as "that which in a given state of information (real or feigned) we do not know not to be true" (Peirce, *The logic of relatives* 206), so anticipating intuitively (not axiomatically) the notion of "local truth" in modal logic. In other terms, for Peirce, the pragmatist is committed to a *strong modal realism* by conceiving of objects in terms of predictive general conditional propositions about how they *would* behave under certain circumstances. In this way, Peirce's logic of relations is effectively an anticipation of a pragmatist "formal ontology" (Peirce, *What pragmatism is*), even though this signified for him a shift from the original set-thoretic *algebraic* logic in which he originally was (Lane). However, today, by using the "Non-wellfounded Set Theory" (Aczel; Abramsky, *A Cook's Tour of the Finitary Non-Well-*

[6] On this point, see my work (Basti, *A Formal Approach to the Ontology of Social Beliefs*), and more systematically (Basti, *Ontologia formale: uno strumento per il dialogo*).

Founded Sets), it is possible to offer a formal, set-theoretic, *coalgebraic*, foundation of modal logic and of local truth semantics (Thomason; Venema; Goranko and Otto), so to fulfil such a programmatic vision of Peirce .

Generally, indeed, in the light of the modern transcendental primacy of the knowledge over the language, in the framework of the Fregean "incomplete" linguistic turn, the main relationship between the "standard" set-theoretic semantics of *mathematical logic*, as to the semantics of the *philosophical logic*, is the following. While the former is an "*extensional* (truth-functional)[7] theory of meaning", the latter is an "*intensional* theory of meaning". Using the efficacious synthesis of J. Searle, the "intensional (with *s*) logic" is the "logic of inten*t*ionality (with *t*)", the logic of the "first-person language" of individuals and groups, as Husserl was the first in stating, and in developing, at the beginning of the last century[8]. This is also the core of Searle's criticism to the early *symbolic* "Artificial Intelligence" (AI) research program, as far as based on Turing's "imitation game", and on a purely extensional theory of meaning, erroneously interpreted as a sufficient theory of human cognition (Searle, *Mind, brains and programs*). This criticism is the basis of his essay on the notion of intentionality, in relationship with intensional logics and neurosciences, based on the essential statement that brains are not computers because the formers, differently from the latters, are able to implement intensional, modal logical calculi (Searle, *Intentionality*).

Formally[9], indeed, what generally characterizes intensional logic(s) as to the extensional one(s) is that two fundamental axioms of the *extensional* predicate logic do not hold in the *intensional* predicate logic. Namely:

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[7] It is the "truth evaluation function" of the Fregean extensional semantics, based on the usage of the truth-tables of the logical connectives of the propositional logic.

[8] It is extremely significant for our aims that Husserl's first development of the *intensionale Logik* or *Inhalt Logik,* in explicit opposition with a *rein extensionale formal Logik* is in the context of his essay on the deductive calculus in formal logic (Husserl, *Der Folgerungskalkuül und die Inhaltslogik*). This paper is in strict relationship with his critical review of Schröder's *Lectures on the Algebra of Logic,* published by Husserl in the same year, 1891 (Husserl, *Besprechung: Schröder, Ernst, 'Vorlesungen'*).

[9] A systematic deepening of the relationship between intensional logic and the metaphysics of intentionality can be found in a famous essay by E. Zalta (Zalta). He is the founder and the principal editor of the well-known *Stanford Encyclopedia of Philosophy.*

1. The *extensionality axiom* ("it is true that: if two classes are equivalent, they are identical"), reducing class *identity* (=) to class *equivalence* ($\leftrightarrow$), i.e., $\mathbf{A} \leftrightarrow \mathbf{B} \Rightarrow \mathbf{A} = \mathbf{B}$, where **A** and **B** are symbols of classes, that is symbols of the extensions of the two respective predicates, *A* and *B*;
2. The *existential generalization axiom* ("it is true that: if a predicate holds for a given individual *a*, there exists at least a generic individual *x* for which the predicate holds"), i.e., $Pa \Rightarrow \exists x\, Px$, where *P* is a generic predicate, *a* is an individual constant, *x* is an individual variable.

For instance, because of the first axiom, if two predicates have the same extension, i.e., they the same class of objects satisfies them, e.g., the predicates "being water" and "being H_2O", they are identical, and then they can substitute with each other, without that the meaning of the proposition changes. Of course, this is true in the scientific usage of the language, but it is no longer true in other usages of language, where the intensional component of meaning are the relevant ones, i.e., where it is critical what the different linguistic communities *intend* with such a term. Indeed, if I substitute "water" with "H_2O" in a poem or in a religious formula, the proposition becomes necessarily meaningless. Moreover, despite "water" is a symbol present in whichever religious tradition as a symbol of "life" and "purification", nevertheless the meaning that each religious tradition attributes to these fundamental religious meanings of water can vary from context to context (e.g., in the Christian Tradition it symbolizes the "water" from the side of Christ crucified, etc.).

The second axiom – effectively a lemma of the precedent one – is another rule of the extensional predicate logic that is fundamental in the scientific language, the so-called "principle of existential generalization". This axiom substantially means that if a given predicate is true for a concrete, singular individual, *a,* it is true also for a generic individual, *x,* of the

same type. A famous philosophical usage of this axiom is the core-principle of the Modern Transcendental of Knowledge, that is, "if I think, then it is true that something is thinking". Where the fundamental mistake of Descartes – rightly emphasized by Gassendi and Kant, for instance – is that he gave an ontological *intensional* interpretation to this inference (i.e., reading it as it was synonym of "if I think, then there exists an individual, immaterial substance that is thinking"). On the contrary, it is trivial for whoever has a sufficient knowledge of logic, that such an inference has an apodictic truth value only and only if we interpret it in an extensional way, i.e., like a *logical tautology* without any "ontological" meaning. In other terms, the statement has an absolute value only *logically* and even *extensionally* (as a tautology), and not *ontologically* and even *intensionally*. I.e., the apodictic value of the statement holds only and only if we identify *a* with *x,* that is, if we consider *a* as equivalent to one of the (infinitely) many generic objects *x* of the domain (extension) of a given predicate, that means, only and only if the fundamental axiom of extensionality holds.

Finally, consider that – as symbolized by the usage of a "truth-functional" conditional symbol "⇒" ("it is true that: 'if…then'"), instead of the simple conditional symbol "→" ("if…then") – both axioms suppose a higher order meta-language as to the object-language they rule. The necessity of attaining to higher order functions/predicates to solve semantical problems is, indeed, the core of the solution of the famous "Russell Antinomy" as to Frege's foundation of arithmetic, proposed in the *Principia Mathematica* by Russel himself through his "type theory".

On the other hand, more generally, the problem that Russell Antinomy addresses in the *Principia* as to Frege's foundations of arithmetic is the core of any semantic antinomy in formal logic and mathematics, as far as based on the recursion principle. That is, as far as based on a *constructive, finitistic* approach to the enumeration of formulas, the Boolean Logic included, as the famous Gödel's two *incompleteness* theorems (Gödel, *Über formal*

unentscheidbare Sätze) for first order formal *languages* definitively demonstrated. A result obtained by Gödel, after having demonstrated that the first order predicate *calculus* is *complete* (Gödel, *Die Vollständigkeit*)[10]. In other words, any consistent set-theoretic semantics in standard set theory can be decided only by supposing *higher order* logics. The higher order predicate calculus is, however, *incomplete*: this is the conundrum of formal semantics, also for Boolean Logic, as far as based on standard set theory. In other terms, the first order predicate calculus is complete, but precisely for this formal strength, the first order theories interpreted as "models", i.e., as an application of the calculus to as many domains of individual objects, are *incomplete*. This implies that formal semantics requires a higher order predicate calculus that unfortunately is *incomplete*. On the other hand, precisely for this weakness of the calculus, the second and higher order theories can be (categorically) *complete*, i.e., *in principle* valid for an infinite number of first order models sharing the same formal structure, as Skolem Theorem demonstrated.

This is the paradox of formal semantics, at least until the development of a coalgebraic modal semantics based on "Non-Wellfounded Sets", and therefore attaining only "local truths". This, by granting consistent first order semantics on a pragmatic basis to Boolean logic formulas – and hence to propositional and predicate logic formulas, because all formally translatable into Boolean logic (numerical) formulas – decrees the superiority of computers as to humans, for the solution of concrete, complex, context-dependent problems. Think at the management of "big data", of traffic control, of internet streams of data, etc.. In these cases indeed, the abstract, infinitistic, way of reasoning of humans mind – i.e., its intrinsic higher order semantics – is *necessarily* destined to fail. Anyway, this is the theoretical core of the actual, growing role of automation in any public and private field of modern society and

[10] Roughly speaking, the difference between a formal *calculus* and a formal *language* is that, a language is a "model", that is a formal calculus applied to a given *domain* (class, set, collection, …) of objects. *Completeness* means that the truth/falsehood of whichever well-formed formula of a given calculus/language can be recursively, finitarily, *decided* as true or false (counted as 1 or 0), on the basis of the explicit rules of this calculus/language.

economy, justifying the definition of our "Semiotic Age" as an "Information Age". Of course, this is becoming progressively true, as far as these theoretical principles apply to the design of new AI devices that in this sense are going further the "Turing Paradigm" in computer science.

Therefore, the main contribution that E. Husserl gave at the end of Modernity to the issue of foundations is that any higher order semantics in formal logic and mathematics is *epistemologically linked* to the human, *self-conscious, intentional* mind. In this way, phenomenology discloses the epistemological *intensional* roots of the same *extensional* logics, *as far as produced by humans*, not in themselves (i.e., as a logical calculus), of course, as we discuss in the next section.

Nevertheless, it is possible to formalize intensional logics and languages – that is, translating them into a *symbolic* language for avoiding ambiguities, and expressing their logical principles into an *axiomatic* way, for granting a rigorous proof of the validity of the argumentations –, just like the extensional logics and languages of pure and applied mathematics. Both scientific and philosophical languages and theories, indeed, after being produced, "live" an independent linguistic life as to the human producers and users[11], so to manifest a "primacy" over the same human knowledge, as the semiotic turn emphasizes. In this way, the formalization also of the modal and intensional logics and languages, underlying universally the philosophical way of reasoning and of speaking within the different cultures and ways of living, is acquiring a growing relevance from many points of view, we discuss in the rest of my contribution. Even though the principal social and cultural contribution consists in the fact that it is the only way for making effectively possible, on rigorous basis, the inter-cultural, and the inter-disciplinary *dialogue,* in our globalization context, against the nihilistic

[11] K. R. Popper spoke evocatively on this regard about the "third world" of theories and languages, beside the first two of the physical and of the mental objects.

and relativistic effects of multi-culturalism.

Anyway, the axiomatic formalization of modal logic, started at the beginning of the XX cent., with the pioneering work of C. I. Lewis (Lewis, *A Survey of Symbolic Logic*; Lewis e Langford, *Symbolic Logic*). As I anticipated, what originally inspired Lewis was his criticism to the usage of the mathematical logic of the *Principia* for the analysis of the philosophical logic, like, on the contrary Wittengstein's *Tractatus* inspired by B. Russell, will do, because Wittengstein at that time did not know Lewis' work, probably also because of Russell disesteem of modal logic in general.

After Lewis' axiomatixation of modal logic[12], however, it became possible to develop a formal approach to the different intensional logics – mainly, "alethic" ("logical" and "ontological"), "epistemic", and "deontic" logics like as many interpretations on different domains of logical modalities (i.e., necessity/possibility in different senses). That is, formally, they correspond to as many semantic interpretations of the axiomatic modal syntax, and of its modal operators of "necessity/possibility"[13].

This early formalization, because it is based on a modal extension of the standard set-theoretic propositional calculus, that is, on an "infinitistic" second-order semantics (Cresswell e Huges; Galvan)[14], makes impossible *in principle* an implementation of this semantics into

[12] Consider that, however, the contemporary recovery of modal logic from the end of XIX cent. on is not related with Lewis' axiomatization of it, but with the pioneering work – apart from Peirce's hints we already discussed – of H. MacColl (1837-1909), who had a relevant explicit influence also over Schröder *Algebra of Logic* (Peckhaus). Indeed, he proposed a non-axiomatic, algebraic interpretation of it in terms of many-valued logic. Afterwards, eminent representatives of the "Polish School of Logic", like J. Łukasiewicz and J. M. Bocheński, shared the same many-valued interpretation of modal logic, in their systematic effort of formalizing the Scholastic philosophy. Anyway, the abandon of the modal logic study during the XV-XIX centuries coincides with the "dark age of philosophy" interpreted as a rigorous science during the Modern Age, just because deprived of its logical *organon*. The preeminence of mathematical science during the Enlightenment has also this explanation.

[13] The literature about these topics is today copious. Let us only quote the more classical textbooks on these topics: (Cresswell e Huges), (Galvan), and the more recent and synthetic (Burgess). For an updated, critical appraisal of the relationship between the phenomenological and the modal approach to intensional logics, see (Wiegand).

[14] The usage of the infinitistic "necessity operator", □, in this formalism, with the meaning "true in all the infinite possible worlds", emphasizes the second order character of this modal semantics. On the other way, the fact that in an algebraic many-valued interpretation of modality, for each new value of truth beside 1/0 it is necessary to add a column to the correspondent truth-table, emphasizes the "finitary" character of this interpretation, given that otherwise we need a matrix with infinite columns. So the usage of the necessity operator is an elegant way for formalizing the possibility of expressing in only symbol all the possible degrees and meanings of necessity/possibility according to the different intensional interpretations of necessity/possibility in different intensional logics. This justifies the success of Lewis' second order axiomatization of modal logic during the XX cent.

the "finitistic" calculations of a "Turing Machine" (TM), so to justify Searle's criticism to classical AI research program[15].

2 Toward a post-modern philosophy: Husserl's formal ontology and the foundation of logic and mathematics

2.1 The Post-Modern Age and the ontological foundation of truth in phenomenology

I developed elsewhere some reflections about the true "paradigm shift" that is happening during the last twenty years in the foundations of physics, mathematics, logic, and of calculus, all related with the complete semiotic turn we are speaking about (Basti, *The quantum field theory (QFT) dual paradigm in fundamental physics*; Basti, *From formal logic to formal ontology*). What we want to deepen here is that all the fundamental topics involved in such a "paradigm shift" are the same that are at the basis of Husserl's deep analysis of the "Crisis" of the European (Modern) notions of the logical, of the mathematical, and of the natural sciences. The difference between the phenomenological and the semiotic solution is that the first one moves inside the Modern Transcendental of Knowing, the second one inside the Post-Modern Transcendental of Language. Indeed, the "naturalistic" and the "objectivist" view of science, as well as the "formalist" approach to the foundations of logic and mathematics, according to Husserl, are at the basis of the contemporary crisis of the European humanism. Moreover, we can add, they are at the basis of the crisis of the Western civilization, not as

[15] This impossibility, of course, does not mean that a programmed computer cannot execute second order calculations. Effectively, several techniques of "second-order programming" are today a standard in computer science, for instance in algorithms for "linear" and "stochastic" optimization problems, or in several algorithms of "automatic theorems demonstrators", etc. Of course, because the programmer is a human subject, the possibility of a second-order programming does not invalidate the thesis of the impossibility for a computer to deal *automatically* with second-order semantic tasks, because of the famous "Turing halting problem" for the UTM. Indeed, a UTM, because in principle able to simulate algorithmically the calculations of all the infinite TM's, is a sort of "second order TM". The way suggested by Turing himself for avoiding the problem is the usage of an "oracle" suggesting every time to UTM the way for halting its computation. In other terms, in "second order programming tasks", it is the human programmer, the "oracle" giving a UTM (our multi-programmable computers) the way for performing a terminating computation that the machine, because of Gödel and Turing theorems, never could find recursively by itself. The notion of "Oracle TM" is the core of the doctoral thesis (Turing, *Systems of logic*) that the same A.M. Turing completed in 1938 at Princeton, under the directorship of another eminent scholar of modern logic and computability theory: A. Church.

such, but in its pretension of leadership as to the rest of the world.

Despite the criticism to such a view is the explicit object of Husserl's last masterpiece firstly published in 1936 (Husserl, *Die Krisis der europäischen Wissenschaften*), nevertheless, as it is evident from the reference to the *transcendental Phenomenologie* in the title of this Husserl's work, such a criticism constitutes the true root of all the phenomenological philosophy. This is true since the very beginning of Husserl's career as a mathematician, and then as a philosopher of mathematics. Particularly, this criticism is about what is the main concern of any foundational theory. That is, the justification of the notion of *truth* in logic and in ontology, both in the *general*, and in the *regional* logics and ontologies of the different mathematical and natural sciences, on one side, and of the philosophical sciences, on the other one.

Effectively, there is apparently an evolution from the most theoretical researches of Husserl about the foundation of truth of the *Logical Investigations,* in which truth depends on the intrinsic relationship between "formal logic" and "formal ontology" as two inseparable components of the idea of "pure logic", and the reflections about truth of the *Crisis.* In it, truth depends on a *pragmatic approach* to ontology based on the *Lebenswelt*. The main thesis of this book is indeed that, despite its constant success, the crisis of the Modern (now, no longer only "European") science consists "in the loss of its meaning for life" (Husserl, *The Crisis of European Sciences* 5). On the contrary, through the "transcendental method of phenomenology", it is possible, according to Husserl, going back to the "primal" role of the "self-evidences of the life-world" (*Lebenswelt*) for the *ontic foundation* of the mathematical and natural-scientific theories. In other terms,

> From objective-logical self-evidence (mathematical "insight", natural-scientific, positive-scientific "insight" as it is being accomplished by the inquiring and grounding mathematician, etc.), the path leads back, here, to the primal self-evidence in which the life-world is ever pre-given. (…) As it is the case in conceiving of geometrical straight lines on the basis of the life-

> world self-evidence of straight table-edges and the like (Husserl, *The Crisis of European Sciences* 128-129).

The Husserlian pragmatic approach to ontology in terms of "forms and contents" of the life-world experiences has its proper, fundamental role in freeing ourselves from "the constant misconstructions which mislead us all, because of the scholastic dominance of objective-scientific way of thinking". Husserl immediately after, synthesizes this predominance of the subjective praxis over the "objectivism" of modern science in the following statement defining

> The objective sciences as subjective constructs – those of a particular praxis, namely, the theoretical-logical, which belongs to the full concreteness of the life-world (Husserl, *The Crisis of European Sciences* 129).

In this way, "the predicative theories of sciences", namely,

> The system of statements meant logically as "proposition in themselves" is rooted, grounded in the life-world, in the original self-evidences belonging to it. Thanks to this rootedness objective science has a constant reference of meaning to the world in which we always live, even as scientists, and also in the total community of scientists – a reference, that is, to the general life-world (Husserl, *The Crisis of European Sciences* 130).

Of course, the scientific theories are not

> Things in the life-world like stones, houses or trees. They are logical wholes and logical parts made up of ultimate logical elements. To speak with Bolzano, they are "representation-in-themselves" ["*Vorstellungen an sich*"], "propositions in themselves", ideal unities of signification whose logical ideality is determined by their *telos*, "truth in itself".
> But this or any other ideality does not change in the least the fact that these are human formations, essentially related to human actualities and potentialities, and thus belong to this concrete unity of the life-world, whose concreteness thus extends farther than that of "things" (Husserl, *The Crisis of European Sciences* 130).

This reference to the problem of truth makes evident that the distance between the *Logical Investigations* and the *Crisis* is more temporal, than theoretical, more apparent, than substantial. Nevertheless, mathematicians and logicians – before all Kurt Gödel, who recom-

mended to logicians and mathematicians the reading, particularly, of the *Sixth Logical Investigation* – can feel themselves more comfortable with the language and the concepts used in the *Investigations*, than with the language and the concepts, more "sapiential" than "scientific", of the *Crisis.* Indeed, while the aim of the *Investigations* is essentially epistemological, the aim of the *Crisis* is essentially anthropological, given the essential thesis of the *Crisis* considering "the history of modern philosophy as a struggle for the meaning of man". Anyway, also in the *Logical Investigations* the issue of the intentional consciousness content, i.e., the "object as such", is central for dealing with the logical (semantic) notion of "meaning", and hence of "truth". In fact, in the *Third Logical Investigation* Husserl defends the *ontological* foundation of the *logical* truths, as far as for him "being/thing" is accessible by knowledge only as "object". Particularly in the "Introduction" to this *Investigation* Husserl refers to the notion of *formal ontology* as the "*pure (a priori) theory of objects as such"*. Therefore formal ontology is

> The systematic place (…) in which we deal with ideas pertinent to the *category of object,* ideas such as Whole and Part, Subject and Quality, Individual and Species, Genus and Species, Relation and Collection, Unity, Number, Series, Ordinal Number, Magnitude etc., as well as the *a priori* truths which relate to these (Husserl, *Logical Investigations. Volume 2* 3).

Effectively, this reference to ontology as "a pure (a priori) theory of objects as such", because of his criticism to the *formalism* typical of the modern "reshaping" of mathematics by the *axiomatic method* (Husserl, *The Crisis of European Sciences* 21-23)[16], constitutes the

[16] In the few pages dedicated by Husserl to such a "reshaping" there is only a very synthetic reference only to "algebra", and henceforth to the "mathematical analysis" (calculus), and to the related issues about the foundation of the "continuum" notion in mathematics. Nevertheless, everybody acquainted with the history of the *axiomatic method* in modern logic and mathematics can "unfold" this Husserlian synthesis, recalling the fundamental steps of this history, lying in the background of the Husserl *Crisis*. This starts from the initial "algebraic" interpretation of "geometry" by R. Descartes, I. Newton and G. Leibniz, by which the millennial problem of the infinitesimal calculus, that blocked the development of the mathematical physics since Archimedes, was finally solved in terms of the modern "mathematical analysis", applied to the Newtonian mechanics. B. Riemann (1854) developed this initial algebrization of geometry to the non-Euclidean geometries, and D. Hilbert, at the end of XIX cent., extended it to the axiomatization of the Euclidean geometry, as well as to the arithmetic of *real numbers*. Finally, mathematicians extended the axiomatic method to the same set-theoretic foundation of logic and mathematics, respectively in the "Zermelo-Fraenkel (ZF)", and in the "Von Neuman-Gödel-Bernays (NGB)" standard set theories, during the first decades of the XX cent. By an axiomatic set theory, it is possible, to solve the problem of the semantics of the non-denumerable sets (e.g., the whole set of real numbers, at the basis of the mathematical analysis) in the foundations of logic and mathematics. On the contrary, a purely algebraic set theory, according to the original research program of the "algebra of logic" by E. Schröder, cannot solve in principle this problem, be-

main motivation of Husserl's phenomenological method and philosophy, since the very beginning of his career. Namely, since his PhD work (1882) in mathematics, concerning the "calculus of variations", Husserl reckons with the algebraic formalism of the modern mathematics (Husserl, *Beiträge zur Theorie der Variationsrechnung*). The "variation calculus" is, indeed, an essential computational tool of the mathematical analysis related to the "perturbation methods" introduced by P. S. Laplace (1749-1827) for extending the "Newtonian mechanics" to the "many body (celestial) mechanics"[17].

Effectively, the perturbation method is a tool of calculus for studying each of the interacting bodies in many body physics as it was "isolated" in the mechanical vacuum, like in the Newtonian mechanics, through the so-called "asymptotic condition" (i.e., by abstractly spacing out the interacting bodies at infinite spatial-temporal distances from each-other, so to cut-off all their interactions). The supposition is that the properties of a physical system do not change, both when the body is interacting with other bodies, as it is *always* in the reality, and when it is not. This, however, happens *only* in the human mind when it abstractly considers a real interacting "thing" as a mind-related isolated "object". In this way, by finding through the asymptotic condition a formal counterpart of the abstractive capacity of the human intentional mind, Laplace obtained the significant result of extending the Newtonian cal-

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cause limited to denumerable sets, as the famous "Löwenheim-Skolem Theorem" (1921) demonstrated. As we see, Schröder's algebra of logic was object of the early (1891) criticism by E. Husserl himself (Husserl, *Besprechung: Schröder, Ernst, 'Vorlesungen über die Algebra der Logik'*). The axiomatic method, however, does not solve the semantic problem by reference to an intentional "subject-object" relationship as in Husserl's attempt, but by distinguishing between a "meta-language" and an "object-language", according to Hilbert's early suggestion. More precisely, the axiomatic set theory solved the foundational problem by moving set-theoretic semantics to a second (and higher) order meta-logic, having first order logic as its own "linguistic object". In this way, it renounced to Schröder's and Peirce's "dream" of a first order semantics for proposition and predicate logic formulas, as far as they are translatable into equation logic formulas for a Boolean algebra. At the same time, the symmetry so constituted between the intentional pair subject/object in epistemology, and the logistic pair meta-language/object-language in formal semantics is at the basis of the reductionist interpretation of the human mind operations in terms of the "Universal Turing Machine" computations. This was the core of the early, symbolic "Artificial Intelligence" (AI) research program, and therefore in the cognitive sciences following the early "functionalist" paradigm, as we see in the concluding Sect. 3 of this paper.

[17] We recall here that the famous Newtonian "gravitational equation", unifying – against Aristotle – the "celestial" and the "terrestrial" physics, is a "two body equation", that is the "gravitational force" is calculated for a system constituted only by two interacting bodies. Any other interaction will thus "perturb" the stability of such a system that makes its equation perfectly *integrable*, i.e., geometrically representable. For this reason, the "perturbative method" introduced by Laplace for solving the problem inaugurated a non-geometrical, purely algebraic, approach to mathematical analysis, based on the systematic usage of algebraic matrices for representing the states of a many body system.

culus in mechanics from geometry to algebra and its matric formalism. This extension is at the basis of all of the most significant successes of theoretical physics during the XIX and XX centuries, culminated into the birth of the so-called "statistical mechanics", until its application into Quantum Mechanics (QM), through the so-called "Feynman diagrams". They are the essential calculus tool at the basis of the "Standard Model" in fundamental physics, and unanimously considered as the best example of application of the perturbation methods in physics.

Therefore, it is not casual that the actual paradigm shift from the QM to the quantum field theory (QFT) of the so-called "physics beyond the Standard Model" [18] depends mainly on the abandon of the perturbation methods, and of their asymptotic condition in quantum calculations (Blasone, Jizba and Vitiello). This condition indeed falsifies the physical reality at the fundamental level, given that each quantum system is intrinsically an "open system", always interacting with the quantum vacuum (QV) fluctuations, intended as a dynamic substrate of force fields connecting everything in the universe.

Now, the new QFT paradigm, conceiving each physical system as the outcome of a web of dynamic relationships having their substrate in the QV is consistent with the ontology of the "semiotic naturalism". It depends, indeed, on the principle of the *transcendental relativity* of everything, as we see in 3.1.

On the other hand, E. Husserl, works inside the Platonic ontology of logic and mathematics, where the logical and the mathematical entities exist before the relations defined on them, and then exist as abstract "objects", because the only relation they have to satisfy for existing in classical mathematics is the *self-identity* relation. That is, a relation only with themselves. So that, he is certainly right in vindicating, since his PhD work on the variation

[18] The Stockholm Royal Academy of Science recently decreed this paradigm shift in fundamental physics last October, by awarding the 2015 Nobel Prize in Physics to T. Kajita and A. B. McDonald for their observational discovery of the neutrino mass. Indeed, "the new observations had clearly showed that the Standard Model cannot be the complete theory of the fundamental constituents of the universe". See, (The 2015 Nobel Prize in Physics - Press Release).

calculus, that the *epistemological* foundation of the main "objects" of mathematics, on which the same algebraic structures apply, that is, the "abstract numbers", requires the reference to an intentional subject.

Therefore, Husserl devoted the following ten years after his PhD to develop the idea of an intentional foundation of the concept of number, culminated into the publication of his *Philosophy of Arithmetic* in 1891 (Husserl, *Philosophie der Arithmetik*). Because this attempt of an intentional foundation of the mathematical logic met with Frege's charge of "psychologism", devastating for a young mathematician like Husserl at that time. this determined the "transcendental turn" of Husserl's phenomenology in his search for a conceptualist ontology of the mathematical and of the logical objects. In this sense, the other two works he published during the same 1891 are very significant, because they testify about the necessity of referring to the intentional subject also in the foundations of the logical and not only of the mathematical calculus.

In his paper about the "deductive calculus", Husserl indeed vindicated the primacy of the "logic of content" (*Inhaltlogik*), that is, of the "intensional logic", for granting a semantic consistency to the extensional calculus of the propositional logic (Husserl, *Der Folgerungskalkuül und die Inhaltslogik*). Henceforth, Husserl's review, (Husserl, *Besprechung: Schröder, Ernst, 'Vorlesungen über die Algebra der Logik. (Exakte Logik). I Band*'), published in the same year, about the first volume of the monumental "algebra of logic" by E. Schröder (Schröder), confirms the same thesis of primacy of the intensional logic. The algebraic "exact logic" of Schröder is, indeed, an extension of the algebraic matric formalism of Boolean Logic to the whole logic, in order to make it "scientific". Its starting point is, indeed, G. Boole's discovery of the possibility of translating consistently the formulas of the propositional calculus and of the monadic predicate calculus into the binary numerical calculus of a Boolean algebra. The vindication of a primacy of the semantic contents as

to the syntactic manipulation of symbols according to algebraic rules is the core of Husserl's criticism to Schröder's "exact logic". And this primacy of the "content logic" over the "algebraic logic" requires for Husserl the necessary reference to a knowing, intentional subject.

What is particularly interesting for our aims is that also Ch. S. Peirce, in a famous review paper of 1897 over the same Schröder's book (Peirce, *The logic of relatives*) that, as we said, inaugurated a new chapter in the history of modern algebra, shared with Husserl the same criticism to the formalism of Schröder's approach. Despite Husserl and Peirce do not know each other, neither know their respective works, their common criticism to Schröder shares also a *pragmatic ontological* and hence *pre-logical* point of view as to the foundation of semantics of predicate logic. Nevertheless, their criticism is made, respectively, from the standpoint of the Modern Transcendental of Knowing [Husserl] – so that the logical symbols are ultimately destined to signify conscious *objects* –, and from the standpoint of the Modern Transcendental of Language [Peirce]. In this case, the logical symbols for signifying something else require only a further third relationship that Peirce generally designed as an "interpretant". In the case of his approach to the algebra of logic, this means that the *fundamental and irreducible relations* on which the algebra of logic depends have to be "triadic", and not "dyadic", as on the contrary, Schröder affirmed. Only, through this third type of relations it is possible to identify any structure similarity between the "sign" and the "signified" on which the "signifying" relationship depends in the algebra of logic.

In other terms, and generalizing, the famous "interpretant" of Peirce's triadic semiotic relationship has not to be necessarily a (knowing) "interpreter" at all. Even though semiotics does not exclude it, in the sense of a usefulness of a semiotic analysis also of the "inner discourse" of the phenomenological experience typical of humans, as I discussed elsewhere (Basti, *For a Post-Modern Ontology. Part I*).

Anyway, because freed from the cumbersome necessity of referring to the conscious-

ness of some knower, Peirce semiosis in several of his works extended itself toward an ontological research program for a *semiotic naturalism*[19], in the sense of an attempt to apply also to physics and to the evolution of the cosmos the triadic categorical analysis of semiotics. That is, a systematic attempt to apply also to nature the famous three categories of "firstness", "secondness", and "thirdness" characterizing Peirce's semiosis.

This further quotation from Peirce's unpublished manuscripts depicts some hints of his *semiotic natural ontology*, because extending his theory of categories also to the natural world, so to suggest a visionary hypothesis of an evolutionary cosmology in physics. This involves the same constitution of the mathematical laws of physics, against the law-like determinism of the classical Newtonian and Laplacian Mechanics, on which Kantian transcendentalism directly depends.

> We have to suppose that in looking into the indefinite past we are looking into back towards times when the element of law played an indefinitely small part in the universe. If the universe is thus progressing from a state of all but pure chance to a state of all but complete determination by law, we must suppose that there is an original, elemental, tendency of things to acquire determinate properties, to take habits. This is the Third or mediating element between chance, which brings forth First and original events, and law which produces sequences or Seconds. Now this tendency to take habits is something essentially finite in amount, an infinitely strong tendency of this sort (unlike an absolute conformity to law) is inconceivable and self-contradictory. Consequently, this tendency must itself have been gradually evolved; and it would evidently tend to strengthen itself. Here is a rational physical hypothesis, which is calculated to account, or all but account for everything in the universe except pure originality itself (Peirce, *One, Two, Three: Kantian Categories*. MS [R] 897).

Nevertheless, the two scientific pillars on which Peirce tried to develop his semiotic evolutionary naturalism were too immature at his time. They are, indeed, an attempt of a dynamic interpretation of the common algebraic structures of statistical physics, as the "secondness" emerging from the "firstness" of the random substrate of nature; and a coalgebraic, co-

[19] According to my knowledge, R. S. Corrington used for the first time this term of "semiotic naturalism", for signifying the systematic effort of Ch. S. Peirce of identifying "the natural enabling conditions of semiosis" (Corrington 89). This led Peirce to extend his famous three categories of "firstness", "secondness", and "thirdness" from the semiotic analysis of language, to the semiotic analysis of nature, as we see immediately.

recursive “unfolding”, of domains of predicative symbols (numerals included) in algebraic logic and mathematics as the necessary *dynamic* “thirdness”. They are all intuitions that, even though without any explicit reference to Peirce’s teaching and vocabulary, are at the basis of the actual paradigm shift in mathematical and natural sciences, and in computer science as well, but also in formal philosophy, having generally in the notions of CT the unifying framework (Basti, *For a Post-Modern Ontology. Part II*).

Nevertheless, against any program of “naturalization of phenomenology”, as we see in the Third Section of this paper, the semiotic naturalism does not negate the epistemological role of phenomenology in the study of *abstract* thought in logic and mathematics, according to the research program Husserl pursued during the largest part of his career as philosopher.

On the other hand, it is evident to everybody the strict relationship existing between the birth of modern Galilean science, and the birth of modern mathematics, on one hand, and the Platonism on the other one. One of the most famous historians of modern science, A. V. Koyré, a follower of the phenomenological school, arrived even at defining the Galilean science as “an experiential proof of Platonism” (Koyré 167). Similarly, A. H. Fraenkel, at the end of the foundational set-theoretic path of modern mathematics according to the axiomatic method (see note 16), stated the strict, indissoluble relationship between standard set theory and Platonism (see Fraenkel, *Set Theory and Logic*).

The intrinsic dependence of the standard set theory on a Platonic ontology of mathematics, consists essentially in supposing that the elements of sets exist in the Universal Collection *V, independently from the relations (“morphisms”, e.g., a function) defined on them. V* – where *V* stays for *Veritas*, i.e., “truth” –, therefore, defines the “universe” of the objects with which a given axiomatic system is dealing with, i.e., the abstract objects “formally existing” in the system. Now, “necessary and sufficient condition” for the

memebership to *V* is that all its members satisfy *a self-identity relationship*[20], a condition stated for the first time, in the history of Western thought, in Plato's Dialogue *Parmenides,* the dialogue in which Plato's metaphysics reaches its most consistent development.

The core of the modern transcendentalism, consists therefore, from Descartes and Kant on, Husserl included, in identifying epistemologically "self-identity" with "self-evidence", so to justify in the usual logical jargon, the denotation of the members of *V* as "objects" (as-to-a-subject) constituting the "universe" of a given axiomatic system.

As an exemplification of such a diffused mentality, let us compare the definition of the *standard* notion of set just at the beginning of Fraenkel's *Abstract Set Theory* book[21], and Husserl's parallel passage about the formal ontology of *independent objects* as "parts of wholes".

Husserl:

> Seen in their mutual interrelations, contents presented together on any occasion fall into two main classes: independent and non-independent contents. We have independent contents wherever the elements of a presentational complex (complex of contents) by their very nature *permit their separated presentation*; we have dependent contents [i.e., "wholes"] wherever this is not the case (Husserl, *Logical Investigations. Volume 2* 6).

Fraenkel:

> *Definition of set.* A set or aggregate is a collection of definite, distinct objects of our intuition or of our intellect, to be conceived as a whole (unity) (Fraenkel, *Abstract Set Theory* 6).

For Fraenkel, indeed, both the "intuition of objects", and "collecting objects into an aggregate" are "intellectual acts" (Fraenkel, *Abstract Set Theory* 6)).

Where the two approaches diverge is about the different logical value we have to attribute to such "primitive objects" of a logical system, and the relative "axioms". Indeed, as

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[20] "*V* is, by definition, the class of all those elements which are self-identical; i.e., since everything is self-identical, *V* is simply the class of all elements" (Quine, *Mathematical logic* 144).

[21] We recall here that A. Fraenkel is co-author with E. Zermelo of the most diffused and used axiomatic set-theory, the "Zermelo-Fraenkel set theory (ZF)" (see note 16).

far as they are self-evident, they are *apodictic* or "absolute" for the modern transcendentalism – from Descartes to Husserl included, passing through Kant, despite the deep differences among these authors. On the contrary, they are only *hypothetical* or "relative" to a limited realm of objects – those of a given mathematical theory – for all the contemporary mathematical logic. For Husserl, indeed, given that we reckon here with the *abstract thought* typical of the mathematical/scientific praxis, as far as related "with the pregnant use of 'think'",

> There is a reference, not to a subjective necessity, i.e., to the subjective incapacity-to-represent-things-otherwise, but to the objectively-ideal necessity of an inability-to-be-otherwise (…) such as to be given in our consciousness of *apodictic self-evidence* (Husserl, *Logical Investigations. Volume 2* 12-13).

In this sense, the difference with the *Crisis* is only in emphasizing that because the "ideal self-evidences of abstract thought" are the result of a particular *praxis* underlying these evidences, that is, the *scientific praxis* of the single scientist, and/or of the scientific community, there exist other most fundamental self-evidences of the *Lebenswelt.* He, indeed, considers this as a sort of "Ur-praxis" of the whole human community. In other terms, in the *Logical Investigations,* all the different objects of the different *material ontologies* suppose their ultimate "objectiveness" (*Gegenständlickeit*), as the *categorical,* ultimate structure of any object to which an intentional act is directed and that is studied by the *formal ontology*[22]. In the same way, in the *Crisis,* the "living experiences" (*Erlebnisse*) of the different "things" emerge onto the common "horizon" of the "perceptual field" constituting the ultimate "ontic" component of the *Lebenswelt*. The link between the two approaches of Husserl research lies, indeed, in the fact that also in the *Logical Investigations* "being" is in continuity with the perception. The "being" intended, both in the *attributive* sense of attributing existence to a thing

[22]In the following passage from the *VI Investigation,* the Husserlian notion of "being" emerges, as well as its "categorical" nature – "the category of being", despite Husserlian categories are different from Kantian ones, because objects of an intellectual (not sensuous) intuition. Nevertheless, from this passage it emerges also why for Husserl, like for Kant and Aquinas, being "is not a *real* predicate", i.e., denoting some property or feature of the objects. "*Not in reflection upon judgements, nor even upon fulfilment of judgements, but in the fulfilment of judgements themselves lies the true source of the concepts State of Affairs and Being* (in the copulative sense). Not in these *acts as objects*, but in *the objects of these acts,* do we have the abstractive basis which enables us to realize the concepts in question" (Husserl, *Logical Investigations*. *Volume 2* 279)

("that white sheet of paper *exists*"), and in the *copulative* sense of expressing the inherence of a property to a thing ("the sheet of paper *is* white"). Nevertheless, this is not the "percept" of some outer or inner sense, because it constitutes the *ultimate fulfilment* of any perceptual presentation of the object to a consciousness.

2.2 The Post-Modern Age and the issue of evidence in the foundation of logic and mathematics

These last quotations express perhaps in the best way the core of a modern interpretation of a Platonic ontology of truth in logic and mathematics, and this explains why Gödel, sharing with Husserl the same ontology and the same criticism to a formalistic approach to semantics, attributed so great importance to Husserl's *VI Investigation*. At the same time, the centrality of the question of "evidence" as to the foundation of truth emphasizes the true motivations of the post-modern "linguistic turn" of the axiomatic method.

The *evidence* – which is clearly a state of consciousness and therefore a property of a subject, either individually, or collectively, or transcendentally intended – is the core of the modern epistemology since Descartes' *Discourse on the Method of Rightly Conducting One's Reason and Seeking Truth in Science* (1637) and Newton's *Treatise on Optics* (1704). Two books synthesizing the turn of modern science and philosophy as to the Aristotelian ones. Indeed, following an original suggestion of J. Maritain, we can synthesize the turn from the Aristotelian to the Modern notion of science as the passage from science as *cognitio certa per causas,* "undoubted knowledge through causas", to science as *cognitio certa per leges,* "undoubted knowledge through laws" (Basti, *L'idea di scienza di Maritain fra passato e futuro*). Now, this turn has in the notion of evidence and in its transcendental role for the foundation of truth the key-point. Also in the Aristotelian sciences – the physical, mathematical and met-

aphysical ones[23]–, indeed, there are laws that are self-evident, but these laws have a *causal* ultimate foundation in the "things" and in their "*real* (causal) relations", not in the mind thinking at them. Even the abstract objects of pure logic and pure mathematics have in the *formal* abstraction from the real quantitative and qualitative properties of things the ultimate foundation of their truth. On the contrary, the *law-like necessity* and therefore the *truth* of the rational, philosophical and scientific modern thought, from the First Rule of Descartes' *Discourse* on, are founded on the conscious "evidences" and their *rational* relations, and not on the things and their *real* relations. The "first rule" of the *Discourse*, indeed, reads: "never to accept anything as true if I didn't have *evident* knowledge of its truth" (Descartes). Namely, it concerns "knowledge" that, either is "evident" because it is a theorem derived from some axiom, or it is "self-evident", because it is an axiom of some deductive procedure.

In other terms, the evidences here concerned are not the "common sense" evidences of the Aristotelian physics and metaphysics that led to clamorous mistakes such as the "geocentrism" in cosmology, justifying the modern "methodic doubt" about all those "old" philosophical certainties. For the "new method" of Galilei and Descartes, indeed, also when these evidences are empirical and not rational, this experience is not the common sense "subjective" experience, but the "objective" experience obtained by *measurements* of physical magnitudes – see Galilei's distinction between "primary", objective because quantifiable, and "secondary" qualities, absolutely subjective, because non-quantifiable. This is the core of the Galilean method, according to which the mathematical laws of mechanics – effectively, the geometrical laws of kinematics intended as the "geometrical science of motions" – are *a priori* with respect to the empirical data. That is, they are not *a posteriori,* because abstracted

[23] As we know, in this list logic is lacking because the discover of the existence of logical laws is the contribution of Stoic philosophers to the history of thought. They are apodictical like the metaphysical ones, but differently from the latter ones, because they are *tautological,* and not because they concern "the being *qua* being". For Aristotle, indeed, who did not develop a propositional, but only a predicate logic with his theory of the categorical syllogism, logic is only a *technique,* a set of rules, an *Organon,* but not a *science*, a set of laws.

from real relations, like in the Aristotelian epistemology.

As everybody knows, the extension of the four *Rules* based on evidence, from the original application to the only abstract realm of mathematics and of geometry, to the self-evidences of metaphysics is the core of the *Discourse* "new method".

Given that – and we must always remember this fact –, the *Discourse* was firstly published as the "Introduction" to an *Essays* volume including not only Descartes' *Treatise on Geometry* – effectively, his fundamental pioneering work on an "algebraic geometry" –, but also his Treatises on *Optics* and *Meteorology. Optics,* indeed, is among all the physical sciences, the physical discipline in which the superposition with the laws of the (Euclidean) geometry is straightforward, because of the rectilinear propagation of the light rays (on short distances)[24]. From this intuition, the Newtonian *Geometrical Optics* derives, so that it is not casual that we find this reference to evidence also in a famous passage of the Newtonian *Treatise on Optics* – quoted also by E. Cassirer as a fundamental source of the Kantian phenomenalism in his monumental treatise on Modern epistemology (Cassirer 402-403).

In this passage Newton exposes the "turn" from the Aristotelian Science, based on the "explanation by causes", to the Modern Science, based on the "explanation by laws". "Scientific explanation", from that moment on, means no-longer finding the causes (real relations) of a given, common-sense phenomenon, but finding a time-independent mathematical law (effectively, a functional relation of the calculus) by which making predictable or retro-dictable some given measurable phenomena. This passage, quoted from the original Ancient English used by Newton and that I preserve, reads:

> These Principles [the laws of Newtonian Mechanics] I consider, not as occult Qualities, supposed to result from the specifick Forms of Things, but as general Laws of Nature, by which the Things themselves are formed; their Truth appearing to us by Phaenomena, though their

[24] On long (astrophysical) distances we have to consider the space-time curvature derived by the gravitation field, so that we have to use a Riemannian geometry, according to the principles of General Relativity, but this is another matter.

> Causes are not yet discover'd. For these are manifest Qualities, and their Causes only are occult. And the *Aristotelians* gave the Name of occult Qualities, not to manifest Qualities, but to such Qualities only as they supposed to lie hid in the Bodies, and to be the unknown Causes of manifest Effects (…). Such occult Qualities put a stop to the Improvement of Natural Philosophy, and therefore of late Years have been rejected. To tell us that every Species ["specific essence" or "nature"] of Things is endow'd with an occult specifick Quality by which it acts and produces manifest Effects, is to tell us nothing: But to derive two or three general Principles of Motion from Phenomena, and afterwards to tell us how [not "why"] the Properties and Actions of all corporeal Things follow from those manifest Principles, would be a very great step in Philosophy, though the Causes of those Principles not yet discover'd: And therefore I scruple not to propose the Principles of Motion above-mention'd, they being of very general Extent, and leave their Causes to be found out (Newton 376-377).

The supposition of self-evidence of the laws of Mechanics justifies therefore the apodictic value attributed by Newton to them (his famous *hypotheses non fingo*), as far as put by him, not only at the beginning of his masterpiece, the *Principia*, but also of the *Opticks*. Its first proposition, indeed, reads: "My Design in this Book is not to explain the Properties of Light by Hypotheses, but to propose and prove them by Reason and Experiments" (Newton 1).

The discovery of the Non-Euclidean Geometries during the first half of the XIX cent., and the consequent discoveries of other branches of the physical sciences that are non-reducible to the Newtonian Mechanics, determined the abandon of the *apodictic* method of the early modern mathematical and physical sciences. They are the Thermodynamics and the Statistical Mechanics, the Quantum Mechanics, and, finally, the Relativity Theory, both Special and General. That is, they determined the passage to the *hypothetical-deductive* method, proper of the modern mathematical and natural sciences in their adulthood.

In other terms, neither one only set of axioms for justifying the existence and the properties of *all* the geometrical objects, nor one only set of principles (physical laws) for justifying the "properties and actions of *all* corporeal things" (as Newton presumed in the pas-

sage of *Opticks* quoted before) *exist.* Consequently, neither the axioms of Euclidean geometry, nor the laws of Newtonian mechanics are *self-evident* and hence *absolute* too.

This determined the growing disaffection of modern scientists toward the *evidence* principle, and to the progressive affirmation of the *axiomatic method* in formal logic and mathematics, from the second half of XIX cent. on. It started with the axiomatization of the Non-Euclidean Geometries by B. Riemann, and of the Euclidean Geometry by D. Hilbert, passing through the axiomatization of the arithmetic by G. Peano, and of the mathematical logic by G. Frege, till arriving to the publication of the *Principia* by A. N. Whitehead and B. Russell, at the beginning of XX cent. The consequent publication by B. Russell of L. Wittengstein's *Tractatus*, trying to extend to the analysis of the philosophical language the axiomatic mathematical logic of the *Principia,* decreed officially the "linguistic turn", characterizing our Post-Modern Age – even though theoretically "incomplete", as we know.

Indeed, like such a sightseeing of the history of the foundation issues teaches, the presumed self-evidence of principles in mathematical sciences is effectively *relative* to the different epochs. Think, for instance, at the "fifth postulate" of the Euclidean Geometry in mathematics foundations (Fraenkel, Bar-Hillel and Levy, *Foundations of Set Theory* 85). More generally, evidence is relative to the different cultures in philosophy, as it is immediate in our "globalization" and "multi-cultural" era. This weakness of evidence depends, ultimately, on the presence of what M. Polanyi, following W. Dilthey, defined as the "tacit dimension of knowledge" (Polanyi and Sen, *The tacit dimension*), as an unavoidable dimension of any form of *personal knowledge* (Polanyi, *Personal knowledge*), and then of any "first-person language", to say the same thing on the semiotic side.

The "axiomatic method", on the contrary, through its double component – the *symbolization* of the ordinary language against any ambiguity in the definition of terms, and the *explicit formulation* of a finite and non-contradictory set of axioms and of inference rules –

grants what the "evidence method" cannot grant in principle. Namely, it grants that the inferences made within a given theory, either scientific or philosophical, *derive exclusively and consistently* from the principles (definitions, axioms, and rules) made explicit at the beginning of the theory, so to avoid any ambiguity and inconsistency in the theory itself. In this way, if a given assert of a given theory expressed in the ordinary language (first-person language, either singular or plural) cannot be consistently derived from the explicit principles after the theory formalization, we can immediately correct the mistake with the support of all the scientific and philosophical community. That is, we are able in a fully controllable way, either of eliminating the inconsistent assert, or of updating the axiomatic apparatus of the theory, so to make consistent the assert in the new axiomatic framework. In this way, the axiomatic method shows all its value as an indispensable method, not only in sciences, but also in humanities, for making possible the interdisciplinary and the intercultural dialogue in our Post-Modern Age. Only in this way, we can avoid consistently the *relativism* and ultimately the *nihilism* of the "universal equivalence" depicted by Vattimo (see above 1.2.1) of the multi-culturalism of our global "Communication Age".

As to the axiomatization of the philosophical doctrines, and against the early confusions of Wittengstein's *Tractatus* and of the Neo-Positivism, we recall here that the proper application of the axiomatic method to philosophical doctrines became possible only after the *axiomatization of modal and intensional logics.* That is, after the axiomatization of the logical *Organon* of the philosophical disciplines, as Husserl rightly emphasized since the very beginning of his career (see before). Historically, the beginning of the axiomatization of the "philosophical logic" is practically contemporary to the publication of the *Principia* and of the *Tractatus.*

It depends, indeed, on the pioneering work of C. I. Lewis (Lewis, *A Survey of Symbolic Logic*; Lewis and Langford, *Symbolic Logic*). He first realized, indeed, that the ex-

clusive usage of the *material implication,* and of the related *extensional theory of meaning* of the mathematical logic of the *Principia*, if applied a-critically to the logical analysis of the philosophical doctrines, as afterward the Neo-Positivism movement effectively did, would have devastating consequences. The largest part of philosophical doctrines, before all the metaphysical ones, would result to be *meaningless,* and the philosophical argumentations to be *inconsistent.* The extensional theory of meaning of the mathematical logic is, indeed, systematically insufficient to justify the inner syntax and semantics of the philosophical discourse, as well as, its unavoidable pragmatic component, as far as related with "persons" (*I-we talk*), as we discussed before (see 1.3).

However, Lewis' axiomatization, as far as it is an extension to modal and intensional logics of classical second order set-theoretic semantics of propositional and predicate logic (Cresswell e Huges; Cocchiarella e Freund), share the same advantages and limitations of the second order semantics of the extensional and mathematical logics. Particularly, it is impossible to justify in it the extra-linguistic reference to individual things that is the core of a semiotic naturalism, not only in philosophy, but also in natural science, and in computer sciences. That is, it is not possible in it to extend formal pragmatics to the unconscious behavior of physical and artificial devices. For formal philosophy, this means remaining prisoners inside the "incomplete linguistic turn" characterizing the logistic approach to the Fregean mathematical logic, as well as of the logistic Church-Turing paradigm of computation in theoretical computer science. Moreover, this means in formal philosophy the impossibility of justifying for humans a formal ontology and epistemology of the *natural realism*, that only could give full satisfaction to the original Peirce's program of a semiotic naturalism that leaded him to define himself along his entire career as a "Scholastic realist" (see below 3.1).

The second-order axiomatization of modal and intensional logics was, however, only the first stage in the recent history of axiomatic modal logic, to which many others follow

toward a final (co-)algebraic interpretation of modal logics (Blackburn, De Rijke and Venema). The following steps were the development of S. Kripke's modal relational semantics (Kripke, *Naming and necessity*), and the consequent extension of modal logic to Boolean Algebras (Goldblatt). Finally, with the coalgebraic justification of modal semantics for Boolean Algebras in the CT formalism (Abramsky; Venema; Blackburn and van Benthem), it was possible to justify in formal semantics the notion of *local truth*, in a first-order modal semantics for Kripke models (Goranko and Otto). In this way, it became possible to satisfy S. K. Thomason's research program of reduction of second-order semantics to modal semantics (Thomason). More deeply, however, all this satisfies the pioneering semiotic criticism to the early formalistic approach to the algebra of logic of Schröder by Peirce in terms of the new development of an algebra of relations in the study of the pre-logic conditions of predicate logic. Under the formal condition of their "categorical dual equivalence" that I explained elsewhere (Basti, *From formal logic to formal ontology*; Basti, *From formal logic to formal ontology*), but that I summarize briefly in 3.3, it is possible to justify in CT logic a coalgebraic semantics of algebras, in general, and of Boolean Algebras in particular.

We can now indicate, therefore, the two different perspectives of Husserl and of Peirce in criticizing the formalism of Schröder's algebra of logic, as the divergent starting point between a phenomenological, and a semiotic interpretation of the formal ontology of the logical inferences (logical thought, in the case of humans). Phenomenology aims at justifying the ontology of the *abstract* logical truths of the standard second-order semantics in mathematical and philosophical logic, typical of humans. Semiotics aims at justifying the ontology of *local* logical truths of the modal first-order semantics in mathematical and philosophical logic, for which no knower is required, even though not excluded.

So, for concluding this synthetic survey about the role of evidence in the history of modern science, we can agree with the mathematician and the philosopher of mathematics H.

Wang[25] when he states that

> There seems to be a relative character in the nature of evidence. What is viewed as evident at one stage of the intellectual process may lose its intuitive evidence at a more advanced stage.

In this way,

> The question of consistency and in general the question of replacing current methods by other ones, can be profitably studied by the use of formal systems. Formalization is not just an end in itself, but in addition a useful instrument that enables us to study the problem of evidence systematically (Wang, *A survey of mathematical logic* 12.17).

This reference of Wang to the possibility of a systematic study of evidence with the help of the axiomatic method, and – we add – in the light of the actual shift from the Transcendental of Knowing to the Transcendental of Language satisfies perfectly Husserl's research program in recognizing a primacy of evidence in science as a *human enterprise*. That is, as a product of the "scientific praxis" of humans. This primacy, in the light of the axiomatic method, is both *genetic* and *theoretic*.

Before all, from the standpoint of the "formalization procedure" of theories as depicted firstly by D. Hilbert as a *Prolegomenon* to his axiomatization of the Euclidean Geometry, his definition – today universally accepted – of the three stages of the formalization of a theory grants a "genetic" primacy to evidence and intuition, i.e., to the *I-we talk* of humans. These stages are, indeed, 1) the *informal or intuitive theory,* by the concept formation and its development; 2) the *formal theory,* by the symbolization and the axiomatization of the former one; 3) the *meta-theory,* proving rigorously consistency and truth, either "abstract" or "local", of the second one.

This primacy becomes "theoretical" after the demonstration of Gödel's two theorems of *incompleteness* of formalized theories. That is, as far as the theories are formalized (I Theorem) and proved (II Theorem) using the *constructive* (recursive), *finitary* methods originally

[25] Wang became famous for his proposal of an axiomatic predicative set theory, and for his studies on the philosophy of mathematics after Gödel fundamental contributions (Wang, *A logical journey. From Gödel to philosophy*).

proposed by Hilbert. Because these fundamental results demonstrated that the second and third steps of Hilbert's formalization procedure could never completely replace the first one as Hilbert's formalistic program pretended, they suggest, on one hand, that the formalization of theories is a sort of never-ending process as to the *intuitive* formulation of them. On the other hand, from the *epistemological* standpoint, they imply the necessity of using higher order meta-languages for proofing consistency and truth of object languages.

Therefore, if we use this hierarchy also for giving meaning to *all* the terms used in the formalized languages, this would imply the unacceptable result of a never ending process. In other terms, the usage of the axiomatic method in formal logic and mathematics is necessary for justifying formally different *mathematical and logical theories* that, for their multiplicity, are necessarily all *hypothetical,* given that no one of them can be the ultimate one, because of Gödel theorems. Nevertheless, the undefined *primitive terms* necessarily used in each of them for defining all the other terms, and for formulating the same axioms and rules of each theory, and from which the *meaning* of all the terms and propositions used in the formal theory depend, have to refer to as many *pure objects* of intuition. With respect to them, therefore, the definitions and axioms of a given theory restrict rigorously their meaning to that used in each theory, so to avoid any falsehood, inconsistency, ambiguity, and misunderstanding in their interpretation. Examples of these primitives are the *general notions* of "number" in the different number theories of formal arithmetic, or of "function" in the different recursion theories, or of "whole" in the different set theories, etc., in mathematical logic. However, other examples are the different primitive notions of "being" in different formal ontologies and metaphysics' – that is the "transcendental name(s) of being" in the Scholastic jargon –, or of "obligation" in different formal ethics, or of "belief" in different formal epistemologies, etc., in formal philosophy. For this reason, Gödel tried to refer since 1959 to Husserl's *pure phenomenology* – effectively, to *VI Logical Investigation* – for deepening the primitive intuition of

"pure object" underlying this apodictic ultimate level of formal enquiry.

3 Toward a post-modern philosophy: the transcendental of language and the semiotic ontology

3.1 The principle of "transcendental relativity" as the core of a Post-Modern ontology

Before sketching briefly something about the modal notion of "local truth" in the framework of CT logic, in its application to QFT both in fundamental physics and in cognitive neuroscience so to fulfill Peirce's "dream" of a semiotic naturalism" in ontology, let us give some hints about the *historical background* of the "semiotic turn" characterizing post-modernity. There is often, indeed, the erroneous conviction that the origins of "semiotics" coincide with Peirce's work, in its opposition versus the modern transcendentalism, based on the primacy of knowledge over language. On the contrary, like Peirce himself was aware, his ontology is explicitly linked, from the very beginning of his research, to the anti-nominalist *realism* of the Scholastic tradition, before all and explicitly, against the Hegelian transcendental idealism in metaphysics (Peirce, *Nominalism versus Realism*)[26]. This self-designation of Peirce as a "Scholastic realist" spans all the research of Peirce, till the end of his research, when he vindicated for his "pragmaticism"[27] the necessity of sharing the Aristotelian modal realism, particularly its doctrine about the existence of "generals" (the "natural kinds" of the semiotic naturalism included), denoted by common names, and not only the existence of individuals, denoted by proper names.

In a word, for Peirce, "the principle of pragmaticism is the Scholastic doctrine of

[26] For Peirce indeed, also the Hegelian "transcendental idealism" is a form of "nominalism". A thesis that is difficult to justify, anyway.

[27] Peirce uses in his last works this term for distinguishing his position from other position about pragmatism such as W. James' one.

realism" (Peirce, *Issues of pragmaticism* 492)[28]. At the same time – and this relates immediately his position to our previous discussion about the Platonic ontology underlying modern mathematics and standard set theory –, he opposed fiercely any interpretation of realism in Platonic terms, as in the classical diatribe about the status of universals between nominalism and realism.

> The notion that the controversy between realism and nominalism had anything to do with Platonic ideas is a mere product of imagination which the slightest examination of the books would suffice to disprove (Peirce, *Review of Fraser's 'The Works of George Berkeley'* 454).

Therefore, evidently, the general relational ontology of Peirce involves also and primarily his anti-Platonic ontology of the mathematical objects (Peirce, *The logic of relatives*). This ontological and epistemological position explains, ultimately, the difficulty Peirce encountered during all his life with the world of mathematicians of his time, B. Russell included, blocking his Academic career, and determining the economic problems he had during all his life.

Anyway, all this confirms J. Deely's individuation in the development of the *semiotic ontology* by John Poinsot during the XVII, that we sketch in the next sub-section, the "lacking ring" of the theoretical chain connecting the Scholastic ontology, based on the metaphysical principle of the *transcendental relativity* of all beings (natural, rational, and linguistic entities), to Peirce's visionary work, and justifying my interpretation of our Semiotic, Post-Modern age, as synthesis-overcoming of the dialectic opposition between the Classical (Middle-Age included), and the Modern Ages.

The notion of *transcendental relativity* of being is introduced by J. Deely, who very appropriately links this notion with Aristotle's solution of Parmenide's "third paradox of

[28] See also other significant passages on these topics present in many Peirce's works quoted in (Lane). Effectively, the link of Peirce with Middle Age Scholastic thinkers is not limited to the Aristotelian ones like Aquinas, but overall with Duns Scotus and his interpretation of modality in ontology and in language. This allows E. C. Moore to conclude: "his [Peirce's] pragmatism (…) is in the main, a reaffirmation of Thomas Aquinas and Duns Scotus concerning the status of universals" (Moore 406).

being", and all deriving from the fundamental axiom of his ontology – "every being exists and cannot be non-existing". This paradox – after the first two, of the contradictory nature of being *multiplicity* (solved by Democritus), and of being *differences* (solved by Plato) –, concerns the contradictory character of *becoming,* if considered as a passage from being to not-being (and vice versa). The Aristotelian solution of such a paradox consists in justifying the becoming as a passage between two *real* (i.e., causally justified) modalities of existing, *actually* and *potentially,* and not between the existence and the not-existence of a given.

> Here is Aristotle's answer finally to Parmenides. When we say that what is, is what it is, we need to understand that is what it is not only actually but at the same time potentially. Things are *both* what they are now *and* what they could be under other circumstances. And, since circumstances are always changing, so is being. In the Latin Age, account would be taken of this fact by saying *that the individual exists relative to its environment*, and this ordering or "transcendental relativity" is part and parcel with the individual's being (Deely, *Four Ages of Understanding* 72. Italics mine).

It is impressive how this passage constitutes also a *descriptive ontology* of the core notions of the fundamental physics deriving from quantum field theory (QFT) as a "thermal field theory", where no physical body is *isolated* in the mechanical vacuum. In other terms, all the physical systems and their stability in time depend dynamically on their interactions with the environment (thermal bath), and not from their insulation from the environment like in Modern Mechanics, that is, their being considered as abstract "objects" for a mind, according to the modern transcendentalism. Because QFT mathematical formalism can be fully justified only in CT logic and mathematics, let us sketch briefly the core of CT in mathematical logic. This will help us to better understand the fundamental contribution of clarification given by Poinsot to the relationship between the Modern Transcendental of Knowledge and the Post-Modern Transcendental of Language.

3.2 A sketch of Category Theory applied to the formal ontology of the semiotic naturalism[29]

The core of CT logic in foundations of logic and mathematics is its anti-Platonic stance that put also logic and mathematics into the framework of the "transcendental relativity" principle so to complete the linguistic turn also in this realm (see 1.3.1 and (Basti, *From formal logic to formal ontology*).

The starting point of such a logic as to set theory is that the fundamental objects of CT are not "elements" but "arrows", in the sense that also the set elements are always considered as domains-codomains of *arrows* or *morphisms* – in the case of sets, domains-codomains of *functions.* In this sense, any object *A, B, C,* characterizing a category, can be substituted by the correspondent *reflexive morphism* $A \rightarrow A$ constituting *a relation identity* Id_A. Morover, for each triple of objects, *A,B,C,* there exists a *composition map* $A \xrightarrow{f} B \xrightarrow{g} C$, written as $f \circ g$ (or sometimes: f ; g), where B is the codomain of f and domain of g[30]. Therefore, a *category* is any structure in logic or mathematics with structure-preserving morphisms. E.g., in set theoretic semantics, all the models of a given formal system because sharing the same structure constitute a category. In this way, some fundamental mathematical and logical structures are as many categories: **Set** (sets and functions), **Grp** (groups and homomorphisms), **Top** (topological spaces and continuous functions), **Pos** (partially ordered sets and monotone functions), **Vect** (vector spaces defined on numerical fields and linear functions), etc.

Another fundamental notion in CT is the notion of *functor, F,* that is, an operation mapping objects and arrows of a category **C** into another **D**, *F:* $\mathbf{C} \rightarrow \mathbf{D}$**,** so to preserve compositions and identities. In this way, between the two categories there exists a *homomorphism up to isomorphism*. Generally, a functor *F* is *covariant,* that is, it preserves arrows directions

[29] For deepening the CT notions we sketch here, please refer to useful introductions such as (Abramsky and Tzevelekos; Awodey).
[30] We recall that typical example of function composition is a recursive, iterated function: $x_{n+1} = f(x_n)$, typical of Boolean Logic.

and composition orders[31], i.e.:

$$\text{if } f: A \to B, \text{ then } FA \to FB; \text{ if } f \circ g, \text{ then } F(f \circ g) = Ff \circ Fg; \text{ if } id_A, \text{ then } Fid_A = id_{FA}.$$

However, two categories can be equally homomorphic up to isomorphism if the functor *G* connecting them is *contravariant,* i.e., *reversing* all the arrows directions and the composition orders, i.e. *G*: $\mathbf{C} \to \mathbf{D^{op}}$:

$$\text{if } f: A \to B, \text{ then } GB \to GA; \text{ if } f \circ g, \text{ then } G(g \circ f) = Gg \circ Gf; \text{ but if } id_A, \text{ then } Gid_A = id_{GA}.$$

Through the notion of contravariant functor, we can introduce the notion of *category duality*. Namely, given a category **C** and an *endofunctor E*: $\mathbf{C} \to \mathbf{C}$, the contravariant application of *E* links a category to its opposite, i.e.: E^{op}: $\mathbf{C} \to \mathbf{C^{op}}$. In this way, it is possible to demonstrate the *dual equivalence* between them, in symbols: $\mathbf{C} \rightleftharpoons \mathbf{C^{op}}$. In CT semantics, this means that given a statement α defined on **C** α is true *iff* the statement α^{op} defined on $\mathbf{C^{op}}$ is also true. In other terms, truth is invariant for such an exchange operation over the statements, that is, they are *dually equivalent.* In symbols: $\alpha \rightleftarrows \alpha^{op}$, as distinguished from the ordinary equivalence of the logical tautology: $\alpha \leftrightarrow \beta$, defined within the very same category.

An application of this principle is able to give a full formal justification of QFT mathematical formalism (Basti, Capolupo e Vitiello). In it, the "mirroring" (of the degrees of freedom) between a physical system (algebra) and its thermal bath (coalgebra) can be interpreted as the *duality* between the categories of "*q* deformed Hopf" coalgebras and algebras for the contravariant application of the same functor *T* (the so-called "Bogoliubov Transform"). It "mirrors" the coalgebra structure into the algebraic one, so to make them homomorphic, and the whole open system stable far from equilibrium. I.e., ***q*-Coalg(*T*)** $\rightleftharpoons$ ***q*-**

[31] A typical example of application in quantum physics, is the attempt in QM of interpreting thermodynamics within kinematics (Connes e Rovelli)

HAlg(T^*). In other terms, the inversion of the energy arrows between a system and its thermal bath for satisfying the energy balance has thus a mathematical counterpart in the duality of the underlying coalgebraic-algebraic formalism. This has an immediate significance for us when we consider that QFT is the fundamental physics also of cognitive intentional system in neuroscience, as we sketched before in **Error! Reference source not found.** that in the light of the present discussion can be extended also to the "mirror neuron systems" (the neurons of the pragmatic inter-subjectivity) in human and primate brains (Rizzolatti e Sinigaglia).

The relevance for cognitive neuroscience and theoretical computer science of the notion of categorical duality is strightforward when we consider another application of this notion in formal semantics. Indeed, in 1936 M. Stone demonstated a fundamental theorem for Boolean Algebras (and Boolean Logic) according to which each Boolean Algebra is isomorphic with a partial ordered set defined on a particular topological space, the so-called "Stone Space", and were the two relative categories are dually equivalent, i.e., **Stone $\leftrightharpoons$ BAlg.** Now, if we define a Stone Space on NWF sets, it is possible to demonstrate another fundamental theorem for Boolean Logic, according to which there exists a duality between the category of coalgebras defined on such a space and the category of modal Boolean Algebras for the contravariant application of the so-called "Vietoris functor", i.e., **Coalg($\mathcal{V}$) $\leftrightharpoons$ Alg($\mathcal{V}$*)** (Venema 393ff.)**.**

Because it is possible to extend to quantum topological spaces (Abramsky, *Coalgebras, Chu Spaces, and Representations of Physical Systems*), and then to QFT coalgebras (Basti, Capolupo e Vitiello, *Quantum field theory and coalgebraic logic*), the consequence for a formal ontology of the semiotic naturalism is immediate (Basti, *From formal logic to formal ontology*).

In CT terms, if a predicative sentence is ordered at "expressing an ontological truth",

it has to express that, through the "*logical* membership" ($\in$) of a sub-class to a class in the *language* (e.g., "horses are mammalians"), as "mirroring" the *ontic (causal)* co-membership in the modal sense of "admittance" ($\ni$)[32] of a genus as to a species, in the *reality.* That is, "species $\ni$ genus", e.g., "mammalians admit horses". The existence of the "thing(s)" depends indeed on such a real "ownership", that is on such a natural foundation of *natural kinds*.

That is, "necessarily A^* belongs to B^* if and only if B owns, admits A", where "-*" are the logical counterpart (class) of the respective natural kind[33]. In symbols: "$\Box_n$ ($A^* \in B^* \leftrightarrows A \ni B$"). Or, more precisely, by emphasizing the homomorphism condition that must link the two halves of the expression, for making it semantically consistent:

$\Box_n\left(A^* \in B^* \xleftarrow{\rightleftarrows} A \ni B\right)$, where the indexed modal operator symbol "$\xleftarrow{\quad}$" stays for the "functor" sending a given morphism from the right formula to the left formula, and therefore ranging over the two opposed categories to which the two formulas respectively belong. Finally, the notion of "ownership" or "admittance" ($\ni$) instead of the standard set-theoretic "inclusion" ($\subseteq$) emphasizes that in the modal coalgebraic logic, as far as, defined on Non-wellfounded Sets, because there exists no total ordering among sets, (indefinitely) many partial orderings (unfolding) between supersets and subsets are allowed. In other terms, the set inclusions from the same superset can follow different paths, so to originate a typical branching "ancestor-descendants", that is, a structure of "set-inclusion tree". In this sense, "mamma-

[32] With the symbol "$\ni$", which means "owns" in the sense: $A \ni B$ means "B owns, admits A", in modal coalgebraic logic we express the necessary and sufficient *formal condition* making true the *dual* algebraic proposition in Boolean logic "$A \in B$". Where the membership predicate "$\in$" is taken according to the *intensional* meaning of "belonging to", and not according to the *extensional* meaning of "being an element of". I deepened elsewhere this notion (Basti, *From formal logic to formal ontology*).

[33] Indeed, because of coalgebras – since they are not limited to polynomials like algebras where the fundamental theorem of algebras holds – are much more suitable that algebras for representing generally dynamic and computational system as "state-transition systems", ontologically, this gives also a solution to the otherwise unsolved problem, in Kripke's relational semantics. That is, the problem of the existence of *natural kinds* (the denoted objects of common names, such as "horses" or "mammalians" in our example) and of the connected Kripke's and Putnam's *causal theory of reference* (see on this point my previous discussion about these problems in (Basti, *Intelligence and reference*), given that a purely *social* justification of their usage in language – like in Putnam and Kripke – gives only a *conventional* justification of them, that is, common terms do not denote anything existing

lians" admit "horses", "dogs", "apes", according to different, *not superposing* branching paths of superset-subset inclusions. In other terms, it is possible to formalize in coalgebras on NWF sets what is not possible to formalize in standard set theory: the unpredictable branching of evolutionary trees that apply not only in biology, but today also in cosmology. Indeed, "cosmogony is the legislator of nature" (Patton e Wheeler), not the transcendental subject as Kant stated at the beginning of his *Critique on Judgement.* That is, the same physical law depends on the universe evolution, as the visionary text of Peirce we quoted in 2.1 about his semiotic naturalism anticipated.

At this point, having enriched formal philosophy with the contribution of CT logic, we have all that is necessary for understanding the essential contribution of clarification that Poinsot can offer us about the shifting from the transcendental of knowing to the transcendental of language.

3.3 Poinsot's proto-semiotics as ontology of signs, and the semiotic naturalism

As we said, the "semiotic turn" characterizing our Post-Modern Age has in the *De Signis* treatise by J. Poinsot (1589-1644) – better known in the history of philosophy with the name of John of St. Thomas – its proto-semiotic start-point at the down of the Modern Age (Deely, *Four Ages of Understanding*). Of this treatise, J. Deely published a commented bilingual edition in Latin and in English (Deely, *Tractatus de Signis. The Semiotic of John Poinsot*). Poinsot, indeed, offered us at the down of Modern Age, a semiotic interpretation of Scholastic and mainly of Aquinas' logic and ontology, alternative to the conceptualist interpretation of Cajetan's and Suarez that, however, prevailed during the Modern Age. Aquinas contribution is essential for aour aims since to him is due the first clair distinction between "object" and "thing", and therefore the extension of the term "intentional" and derivated from the ethical to the cognitive realm. This is strictly related with Aquinas'

ontological foundation of truth, as far as it is available not in the so-callled “first operation of intellect” when it produces the concept and then knows abstractly the “object”, but in the “second operation” of intellect, when it formulates the judgement. Aquinas’ ontological foundation of truth, from which his epistemological “natural realism” derives, consists thus in affirming that intellect knows truth in judgements, as far as the logical composition subject-predicate “is measured” – not “measuring” like Sophists said – by the ontic (causal) composition species-individual (or genus-species) in nature (Aquinas, *Quaest. De Veritate,* q. 4, art. 1). In other terms, the true sentence must satisfy the *ontological bi-conditional* (⇆)

(Basti, *From formal logic to formal ontology*), according to which a predicative sentence is true only and only if its logical structure “is mirroring” the ontic structure of the thing(s) to which the proposition refers. I.e., the homomorphism (adequacy) between the reality and the intellect, does not concern the intellect as far as it is apprehending “objects”, i.e., the *rational beings,* or the universal concepts of “horse” and of “mammalian” separately. On the contrary, by coming back to senses where the “links” between the referents of concepts are not lost because “before” their abstraction (Basti, *La teoria "complexa" della cogitativa*), the intellect can produce true sentences. In a word, adequacy and hence truth concerns the intellect in producing the “predicative sentence”, that is, as far as it is *composing,* and not simply “combining”, several mental objects (at least two universals, in the case of a categorical predication, like ours) into a “locally true” predicative sentence, because “mirroring” the ontic structure of the referred thing.

The outstanding contribution of Poinsot is to make explicit such an implicit semiotic interpretation of truth in Aquinas. For making this, he enriched for the first time in Western thought, and more than two centuries before Peirce, the ontology by a third category of relations beside the classical ones of the *rational* (logical) and *real* (natural): the *relations in lan-*

guage from which the sign as a "being-*for* something else" is constituted. Poinsot defines such relations as *transcendental relations* for two reasons. Firstly, because they do not belong to the category of relations of the Aristotelian table of categories of which nature is "dyadic", that is they are as many "being-*to* something else". If a semiotic relation is a "being-*for*" this implies the reference to a "third term", i.e., to something "absolute" with respect to the two related ones of a normal dyadic relation. Secondly, because, precisely for their triadic nature of transcendental relations, they can range (*vagare* is the intriguing term used by Poinsot) over all the categories, that is, they can assume whichever predicative meaning in language. The following is the fundamental passage of Poinsot's outstanding "proto-semiotics":

> The *transcendental relation,* that is nothing but a relation in language, has not the principal meaning of "relation" [i.e., it does not belong to the ontological category of "relations"], but of something "absolute" to which some relation can be attributed. Indeed, if it was not implying something "absolute", it would be not "transcendental", that is ranging over different categories (*idest vagans per diversa genera*), but it would belong to only one category (Poinsot, *De Signis,* 578b5-579a7. In: (Deely, *Tractatus de Signis. The Semiotic of John Poinsot* 90)).

If we compare this doctrine with a fundamental passage of Aquinas' realism, it is immediate the cleverness' of Poinsot's hermeneutics of Aquinas:

> If [truth] denotes the "truth of predication" (*veritatem predicationis*), "human" is predicated more truly as to thing existing in its proper nature, than in the way by which [i.e., by self-conscious abstraction] it is in the "mental word" (*verbum mentis*) [i.e., the conceptual object] (Aquinas *Q. De Veritate,* q. 4, art. 6).

In other terms, the *asymmetric logical relation of reference* for which language refers to things, i.e., "language → things", but not vice versa (things do not refer to language), has in the opposite direction of *the causal relations* ("real relation") from things to languages by senses, i.e., "language ← things", their foundation. Now the brilliance of Poinsot's purely relational semiotic analysis consists in individuating in the "reflexive" *rational relations* (↻), by which "*self-referential* objects" for a mind are constituted, the core of the *reversal of the arrows* between things and languages, and therefore, the core of the *semiosis*. However – as it

is evident by Aquinas doctrine that also animals "can behave truthfully" through senses, without being self-consciously aware of the truth of their behavior (their languages included) –, this reflexivity has not to not be "consciously doubled" (see the three dimensional sphere of "object" in Fig. 2), for justifying the truth. That is, the "adequacy to thing" of animal's behaviors/languages.

In other terms, the semiosis consists in the compositionality of these three reciprocally irreducible types of relations. In fact, according to Poinsot's brilliance, not only, effects are not *per se* "signs" of their causes – because real relations are "dyadic" only *ad aliud,* "as-to-other" – but also "objects" are not *per se* "signs" of things – because rational relations are "dyadic", only *ad semetipsum,* "as-to-themselves". Only, through the "transcendental relations" in language, "as expressing mental objects and signifying real things", both, "effects" and "objects" acquire the new ontological dignity of "signs", of *being-for-something-else,* and, consequently, the language acquires in humans its power of referring mental objects to things.

The following scheme of Fig. 1 about the integration of Poinsot's three relations could be therefore useful to us for offering a synthesis of his doctrine.

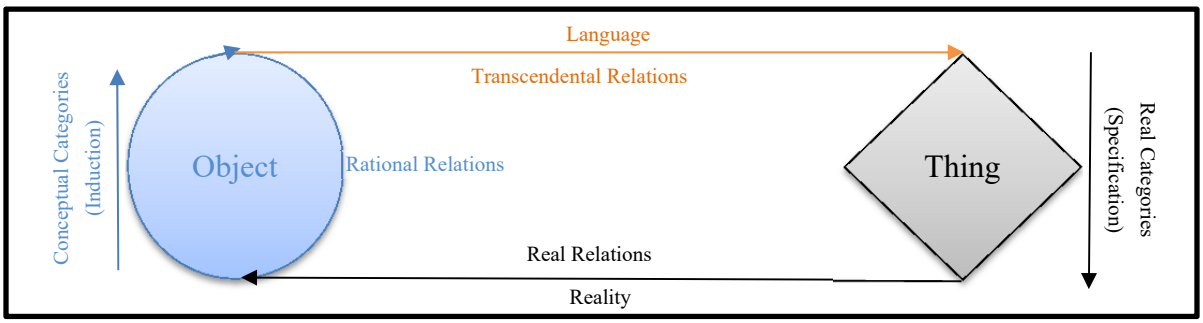

Fig. 1. Scheme of the three types of relations (real, rational, transcendental), and of their composition

4 Conclusions: the Post-Modern Transcendental of Language and the new role of the phenomenological *epoché*

To conclude, it is important to recall the new role that the phenomenological *epoché* has in the new contemporary philosophical and scientific context, after the shifting from the *transcendental of knowing,* to the *transcendental of language*. The *epoché* maintains, indeed, its irreplaceable role to disclose to the phenomenological analysis the "world of objects" characterizing the human experience of the *Lebenswelt* in all its multifarious richness, and expressing itself into the *first person language* – the *I-We Talk or Belief Talk* – of the natural languages of the different human individuals and groups.

The history of the phenomenological movement widely demonstrated that phenomenology is a precious tool for philosophy in order to connect the modern thought, on one side, to the glorious pre-modern traditions – without pretending, however, to reduce to its method such traditions, as we saw with respect to Aquinas, for example –, and, on the other side, to modern sciences intended as human forms of knowledge. That is, the modern sciences considered *not in themselves*, but considered as *human mind products*, and as an essential component, but not unique, of the modern *Lebenswelt* and of the modern rationality. In this way, the *epoché* can continue to play an essential role, generally, as a post-modern sentinel against any reductionist pretension of the scientist ideology, and, specifically, as a precious source of experiential evidences for human sciences. On the other hand, at the beginning of the present XXI cent., after the linguistic paradigm shift concerning not only philosophy, but also the mathematical and natural sciences, for continuing to play its role in our Post-Modern Age, it is essential that phenomenology integrates itself with the semiotic paradigm, in the framework of their complementarity, and reciprocal irreducibility. I tried to show, from many points of view, not only the feasibility of such an integration, but also its fruitfulness.

To sum up, we are faced today with the following "systematic triangulation" of the subjective/intersubjective evidence as expressed into natural languages, in terms, of the formalized talks, either of the mathematical logic, as formalizing our personal way of mathematical reasoning, or of the intensional modal logics, as formalizing our personal way of philosophical reasoning. This *semiotic triangulation* is among: 1) the *I/We-talk* of the intentional thinking of humans and of their extensional and intensional ways of reasoning, as expressed primarily into natural languages of the different cultures. 2) The *O-talk*$_1$ formalizing our extensional and mathematical way of reasoning into the different formal languages, following the rules of the mathematical (algebraic) logic, where no modality is allowed. 3) The *O-talk*$_2$ formalizing our modal and intensional ways of reasoning, into the different formal languages, following the rules of the different (coalgebraic) modal intensional logics (i.e., according to different modalities and degrees of necessity: "ontic", "epistemic", "deontic"). Both, *O-talks* however are useful for modeling the logical (computational) behavior of physical natural and artificial "communications agent.

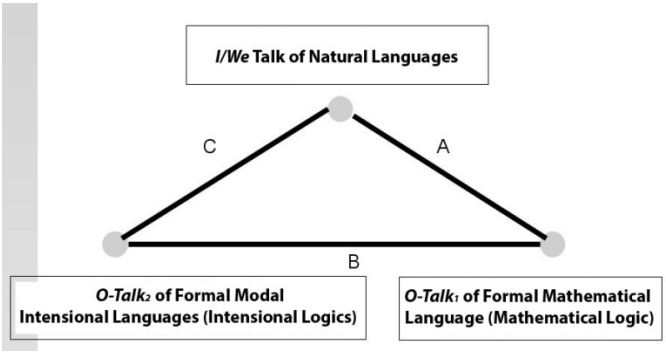

Fig. 2. The semiotic triangle between natural and formal languages, either extensional (mathematical) or intensional (philosophical).

Such a semiotic triangulation is, finally, essential for understanding how the present and the future of the philosophy in our semiotic age is strictly related to a formalization of the different humanistic disciplines, as an essential tool of inter-cultural (among different humanisms), and inter-disciplinary (between humanities and sciences) dialogue, for emphasizing

differences and contact points on a solid basis. Only in this way, indeed, the richness of the philosophical speculation, inside of the different philosophical traditions, might continue to play its essential critical and anthropological role also in our Post-Modern Age. For avoiding that, the nihilist destiny of the dissolution in our Communication Age of the human inter-changes into "the indefinite transformations of the universal equivalence" – according to the remembered efficacious synthesis of G. Vattimo – be our destiny in the present, and in our future.

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Keywords

Semiotics, Phenomenology, Intentionality, Mathematical Logic, Extensional Logic, Modal Logic, Intensional Logic, Formal Philosophy, Formal Ontology, Set Theory, Category Theory, Coalgebras, Boolean Algebra, Computer Science, Quantum Mechanics, Quantum Field Theory.

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